Logical Universality from a Minimal Two-Dimensional Glider Gun

José Manuel Gómez Soto
Department of Mathematics
Autonomous University of Zacatecas
Zacatecas, Zac. Mexico
jmgomez@uaz.edu.mx
http://matematicas.reduaz.mx/~jmgomez

Andrew Wuensche
Discrete Dynamics Lab
andy@ddlab.org
http://www.ddlab.org

To understand the underlying principles of self-organization and computation in cellular automata, it would be helpful to find the simplest form of the essential ingredients, glider guns and eaters, because then the dynamics would be easier to interpret. Such minimal components emerge spontaneously in the newly discovered Sayab rule, a binary two-dimensional cellular automaton with a Moore neighborhood and isotropic dynamics. The Sayab rule's glider gun, which has just four live cells at its minimal phases, can implement complex dynamical interactions and the gates required for logical universality.

Keywords: universality; cellular automata; glider gun; logical gates

1. Introduction

The study of two-dimensional (2D) cellular automata (CAs) with complex properties has progressed over time in a kind of regression from the complicated to the simple. Just to mention a few key moments in cellular automaton (CA) history, the original CA was von Neumann's with 29 states designed to model self-reproduction, and by extension—universality [1]. Codd simplified von Neumann's CA to eight states [2], and Banks simplified it further to three and four states [3, 4]. In modeling self-reproduction it is also worth mentioning Langton's "Loops" [5] with eight states, which was simplified by Byl to six states [6]. These 2D CAs all featured the five-cell von Neumann neighborhood.

Another line of research was based on the larger 9×9 Moore neighborhood. Conway's famous Game of Life binary CA [7, 8] featured the first emerging gliders, and Gosper was able to devise "glider guns" to fire a stream of gliders. Interactions involving glider streams

https://doi.org/10.25088/ComplexSystems.27.1.1

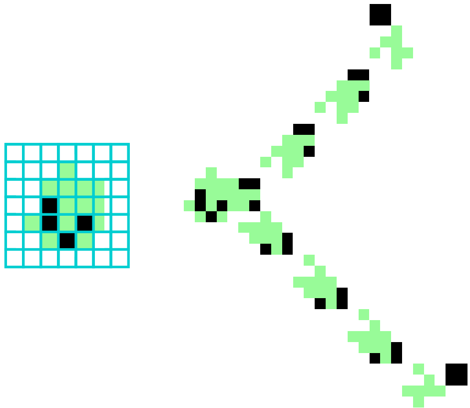
and "eaters" enabled the demonstration of universal computation. A few "Life-like" CAs featuring glider guns were subsequently discovered that follow the Game of Life birth/survival paradigm [9].

More recently, CAs that feature glider guns, but not based on birth/survival, have been found, including Sapin's R rule [10] and the authors' X rule [11] and precursor rule [12]. Glider guns have also been discovered in CAs with six- and seven- cell neighborhoods on a hexagonal 2D geometry with three values [13, 14]. From this we can see that the architecture of CAs that are demonstrably able to support emerging complex dynamics is becoming simpler—arguably a positive development, since a minimal system becomes easier to interpret. This is important if the underlying principles of universal computation in CAs are to be understood, and by extension the underlying principles of self-organization in nature.

The essential ingredients for a recipe to create logical universality in CAs are gliders, glider guns, eaters and the appropriate diversity of dynamical interactions between them, including bouncing and destruction. Of these, the glider gun or "pulse generator," a device that ejects gliders periodically, is the most critical and elusive structure. To some extent, glider guns have been demonstrated in one dimension [15, 16], and to a lesser extent in three dimensions [17], but here we consider the more familiar and much more studied 2D space, which is also easier to represent and manipulate. Glider guns in 2D CAs suitable for computation usually comprise extensive periodic structures. For example, Gosper's Game of Life glider gun [7] maintains about 48 live cells. Here we present a very small glider gun that emerges spontaneously in the newly discovered Sayab rule, named after the Mayan-Yucatec word for a spring (of running water).

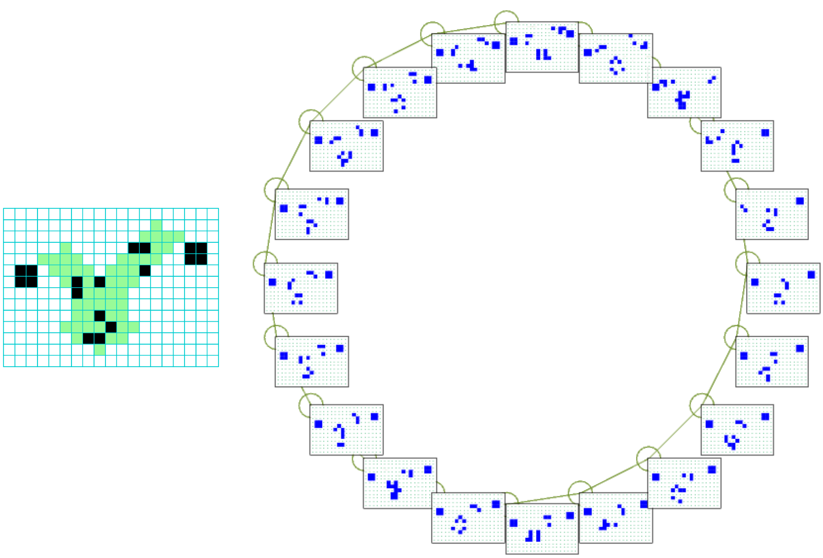
The Sayab rule is a binary 2D CA with a Moore neighborhood and isotropic dynamics (Figure 1). Though analogous to the Game of Life and the recently discovered precursor rule, the Sayab rule has a glider gun consisting of just four live cells at its minimal phases, as well as eaters and other essential ingredients for computation (Figure 2). We show that the Sayab rule can implement a diversity of complex dynamical structures and the logical gates required for logical universality and supports analogous complex structures from the Game of Life lexicon—still lifes, eaters, oscillators and spaceships. (We designate a CA "logically universal" if it is possible to build the logical gates NOT, AND and OR, to satisfy negation, conjunction and disjunction. "Universal computation" as in the Game of Life requires additional functions [7, 18], memory registers, auxiliary storage and other components.)

This paper is organized into the following further sections: 2. The Sayab Rule Definition, 3. Glider Guns, Eaters and Collisions, 4. Logical Universality and Logical Gates and 5. Concluding Remarks.



(a) (b)

Figure 1. (a) One of the Sayab rule's minimal glider gun patterns, of four live cells. (b) The glider gun GG1 in action, shooting two diagonal glider streams with a frequency of 20 time steps and glider spacing of five cells. Each glider stream is stopped by an eater. Because the system is isotropic, any orientation of the glider gun is equally valid. Green dynamic trails are set to 10 time steps. Note: Green dynamic trails mark any change on a zero (white) cell within the last 10 time steps, giving a glider a green trailing wake. Ten time steps is the setting in all subsequent figures with green dynamic trails.



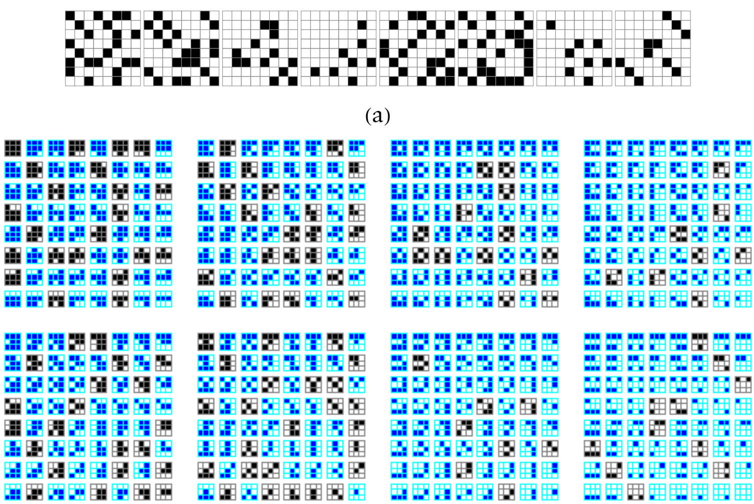
(a) (b)

Figure 2. The Sayab rule glider gun attractor cycle [19] with a period of 20 time steps composed of two phases, where opposite glider gun patterns are

flipped. The direction of time is clockwise. A small patch was isolated around a glider gun by two close eaters. (a) A detail of a patch with a minimal glider gun of four live cells (green denotes change), alongside (b) the same pattern on the attractor cycle.

2. The Sayab Rule Definition

The Sayab rule is found in the ordered region of the input-entropy scatter plot [20] close to the precursor rule [12] and from the same sample and shortlist [11, 12]. The input-entropy criteria in this sample followed "Life-like" constraints (but not birth/survival logic) to the extent that the rules are binary, isotropic, with a Moore neighborhood, and with the λ parameter [21], the density of live cells in the lookup table, similar to the Game of Life where $\lambda = 0.273$. Isotropic mapping—the same output for any neighborhood rotation, reflection or vertical flip—reduces the full rule table (Figure 3) with $2^9 = 512$ neighborhood outputs to just 102 effective outputs [22], from which just 29 symmetry classes map to 1 (Figure 4).



(a)

(b)

Figure 3. (a) The Sayab rule table based on all 512 neighborhoods and (b) expanded to show each neighborhood pattern. 131 black neighborhoods map to 1; 381 blue neighborhoods map to 0. Because the rule is isotropic, only 102 symmetry classes are significant, as described in Figure 4.

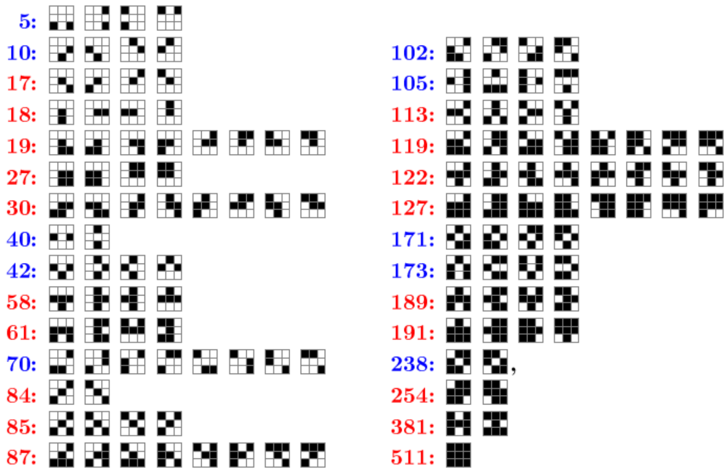


Figure 4. The Sayab rule's 29 isotropic neighborhood symmetry classes that map to 1 (the remaining 73 symmetry classes map to 0, making 102 in total). Each class is identified by the smallest decimal equivalent of the class, where the 3×3 pattern is taken as a string in the order $\begin{smallmatrix}876\\543\\210\end{smallmatrix}$—for example, the pattern  is the string 001110111 representing the symmetry class 119. The class numbers are colored depending on the value of the central cell to distinguish birth (blue) from survival (red), but no clear "Life-like" birth/survival logic is discernible.

3. Glider Guns, Eaters and Collisions

From the Game of Life lexicon, we borrow the various names for characteristic dynamical patterns or objects, including glider guns, gliders, eaters, still lifes, oscillators and spaceships. A glider is a periodic mobile pattern that recovers its shape but at a displaced position, making it move at a given velocity, sometimes referred to as a mobile particle. A glider is usually identified as moving on the diagonal, whereas an orthogonal glider is called a spaceship. A glider gun is a periodic pattern in a fixed location that sends, shoots or sheds gliders into space at regular intervals.

In the Sayab rule, the spontaneous emergence of its basic glider gun, as well as isolated gliders, is highly probable from a sufficiently large random initial state (Figure 8) because the four glider patterns (Figure 5) are very simple and likely to occur or emerge by chance—likewise, the smallest glider gun patterns (Figure 9). Simple still lifes (Figure 6) and oscillators (Figure 7) (which may act as eaters that destroy gliders but remain active) are also likely to occur or emerge from random patterns. The basic glider gun is also probable in subsequent evolution because it can result from the collision of two gliders, or a glider and an oscillator, though the glider gun can also be destroyed by incoming gliders and other interactions.

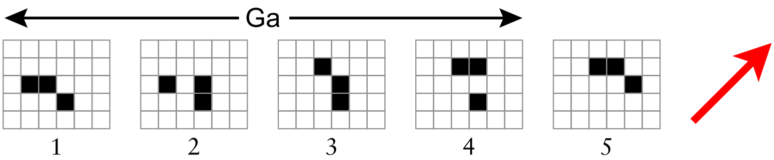


Figure 5. The four phases of the Sayab rule glider Ga, moving NE with speed $c / 4$, where c is the "speed of light;" in this case, for a Moore neighborhood, c equals one cell per time-step, diagonally or orthogonally.

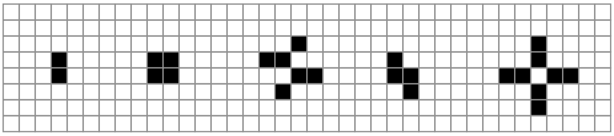


Figure 6. Examples of still lifes.

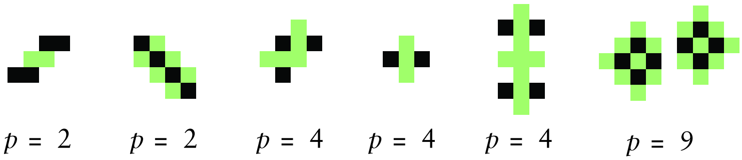


Figure 7. Sayab rule oscillators with the periods indicated.

As can be seen in its attractor [19] (Figure 2), the Sayab rule's basic glider gun GG1 (Figure 1) has a core that varies between just four and 11 live cells during its cycle of 20 time steps, which is composed of two equivalent phases of 10 time steps. After 10 time steps, the core patterns are reversed. In Figures 2 and 9, the core and its twin 45° glider streams face toward the north, but the glider gun can be oriented to face in any of four directions. The glider gun shoots gliders at intervals of 20 time steps with a speed of $c / 4$, and a glider takes 20 time steps to traverse five (diagonal) cells, which is also the spacing of gliders in a glider stream. This spacing can be doubled (without limit) by combining the basic glider guns into compound glider guns (Figures 16 and 17).

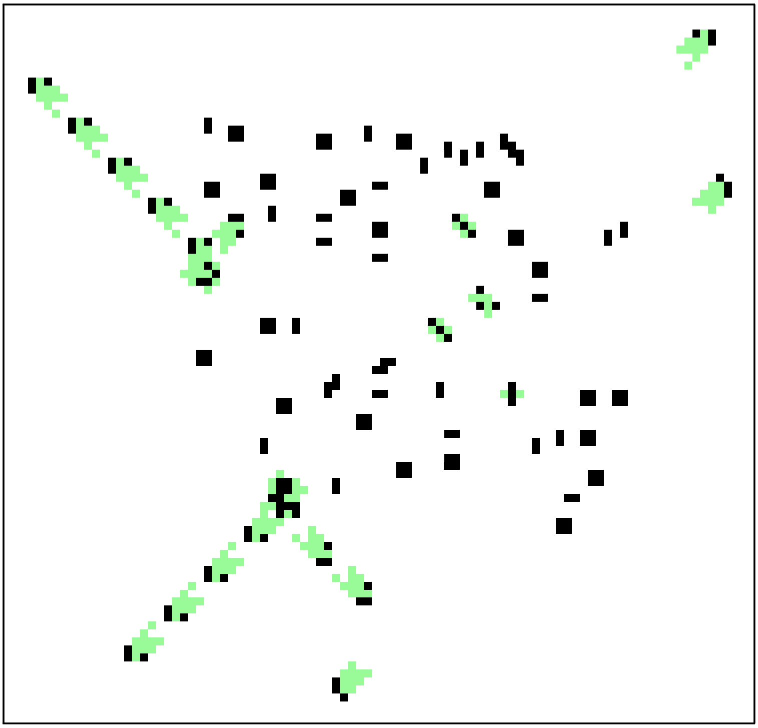


Figure 8. A typical evolution emerging after 108 time steps from a 50×50 30% density random zone. Two stable glider guns have emerged, together with other gliders, still lifes and oscillators.

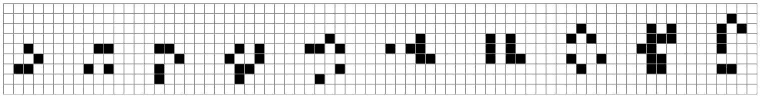
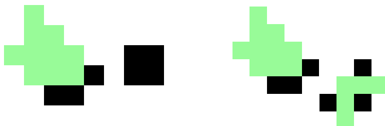


Figure 9. The glider gun core for 10 successive time steps—in the next 10 time steps the same reversed patterns are repeated, to make the period 20 attractor cycle (Figure 2). The pattern sequence is from left to right. Any of these patterns are the seeds of a glider gun, with the smallest, four live cells, being the most probable to occur in a random pattern.

In the Sayab rule, there are many possible outcomes resulting from collisions between two (or more) gliders, and between gliders and still lifes or oscillators. These have been examined experimentally but not exhaustively. The outcomes depend on the precise timing and points of impact and can result in the destruction, survival or modification of the various colliding objects. For the purposes of this paper, we highlight some significant collision outcomes.

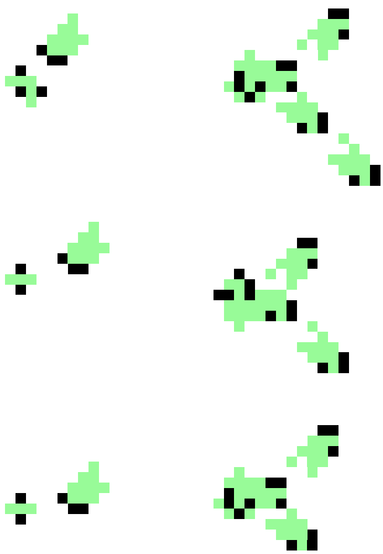
Eaters that are able to stop a stream of gliders are a necessary component in the computation machinery. They can be derived from still lifes or oscillators (Figure 10). The glider gun itself can be the outcome of a collision between a glider and an oscillator (Figure 11) or between two gliders (Figure 12).

A particular but not infrequent collision situation can arise between a stream of gliders and an oscillator, which results in a retrograde stable pattern moving backward, a sort of footprint. This eventually destroys the originating glider gun, as illustrated in Figure 13.



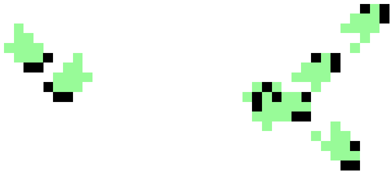
(a) (b)

Figure 10. Collisions between a glider and an eater, (a) derived from a still life and (b) from an oscillator.



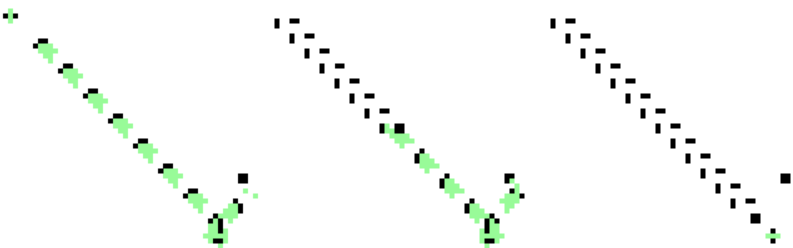
(a) (b)

Figure 11. (a) Three different collisions between a glider with an oscillator create a glider gun (b) shown after 43 time steps.



(a) (b)

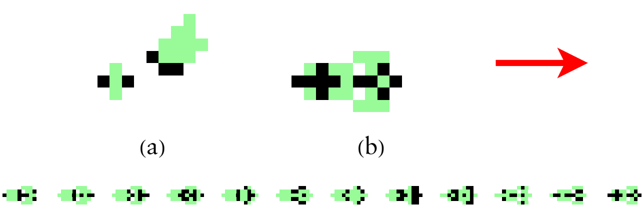
Figure 12. (a) Two gliders colliding at 90° create a glider gun (b) shown after 48 time steps.



(a) (b) (c)

Figure 13. Glider gun stream (a) collides with an oscillator, resulting in a retrograde stable pattern (b) moving backward that eventually destroys the glider gun (c).

A small, slow-moving spaceship (an orthogonal glider) can result from a collision between a glider and an oscillator, as shown in Figure 14. The spaceship that emerges has a frequency of 12 and speed of $c/12$, so it takes 12 time steps to advance one cell. Larger spaceships with various frequencies are shown in Figure 15.



(a) (b)

(c)

Figure 14. (a) A glider collides with an oscillator. (b) After 25 time steps, a slow-moving spaceship is created moving east. (c) The 12 phases of the spaceship.

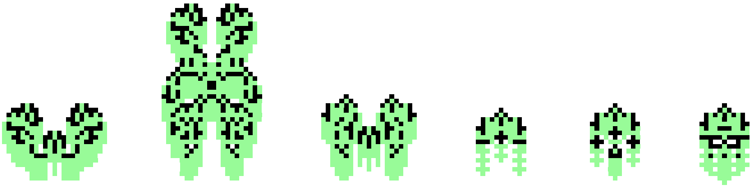
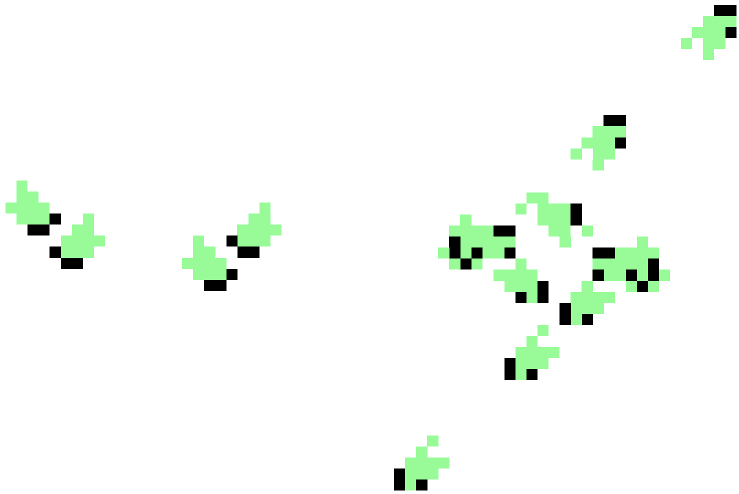


Figure 15. Six large spaceships moving north with speed $c/2$. Periods, from left to right, are 2, 2, 2, 4, 4, 4.



(a) (b)

Figure 16. (a) Two pairs of gliders, each pair colliding at 90°, form a preimage of GG2. (b) The compound glider gun GG2, shown after 138 time steps, shoots gliders with a frequency of 40 time steps, and glider spacing is 10 cells.

A compound glider gun (GG2) can be built from two interlocking GG1 glider guns. GG2 shoots two glider streams in opposite directions with a frequency of 40 time steps and a glider spacing of 10 cells (twice GG1). The dynamics depend on glider streams colliding at 90°, resulting in the destruction of one glider stream, and alternate gliders in the other glider stream. Collisions leave behind a sacrificial "eater" that destroys one of the next pair of incoming gliders.

Two GG2 glider guns can be combined into a larger compound glider gun (GG4, Figure 17) where analogous collisions result in doubling the GG2 frequency and spacing, so the GG4 glider stream has a frequency of 80 time steps and spacing of 20 cells. This doubling of glider-stream frequency and spacing with greater compound glider guns can be continued without limit.

Figure 17. The compound glider gun GG4 shoots gliders with a frequency of 80 time steps, and glider spacing is 20 cells.

4. Logical Universality and Logical Gates

Post's functional completeness theorem [23, 24] established that it is possible to make a disjunctive (or conjunctive) normal-form formula using the logical gates NOT, AND and OR. Conway applies this as his third condition for a cellular automaton to be universal in the full sense. The three conditions, applied to the Game of Life [7], state that the system must be capable of the following:

1. Data storage or memory.
2. Data transmission requiring wires and an internal clock.
3. Data processing requiring a universal set of logic gates NOT, AND and OR, to satisfy negation, conjunction and disjunction.

This section is confined to demonstrating the logical gates, so Conway's condition 3, for universality in the logical sense. To demonstrate universality in Conway's full sense it would be necessary to also prove conditions 1 and 2. (Alternatively, full universality could be proved in terms of the Turing machine, as was done by Rendell [18].)

We propose that the basic existential ingredients for constructing logical gates, and thus logical universality, are as follows:

1. A glider gun or "pulse generator," that sends a stream of gliders into space (Figures 1 and 2). Gliders are not listed separately because they are implicit in the glider gun.
2. An eater, based on a still-life or oscillator, that destroys an incoming glider and survives the collision, so can stop a glider stream (Figure 10).
3. Complete self-destruction when two gliders collide at an angle. Any debris must quickly dissipate, and the gap between gliders must be sufficient so as not to interfere with the next glider collision (Figure 18).

These ingredients exist in Sayab rule dynamics, where collision outcomes depend on the precise timing and points of impact. GG1 glider gun streams are made to interact with glider/gap sequences representing "strings" of data. Using the correct spacing and phases, the logical gates NOT, AND and OR can be constructed. Examples are shown in Figures 19–21, where gaps in a string are indicated by gray circles, and dynamic trails of 10 time steps are included. Any input strings can be substituted for those shown. Eaters are positioned to eventually stop gliders.

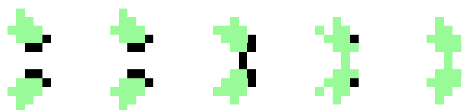


Figure 18. Two gliders colliding at 90° self-destruct. Five consecutive time steps are shown. This is a key collision in making logical gates. Head-on collisions also self-destruct, but are not as useful in this context.

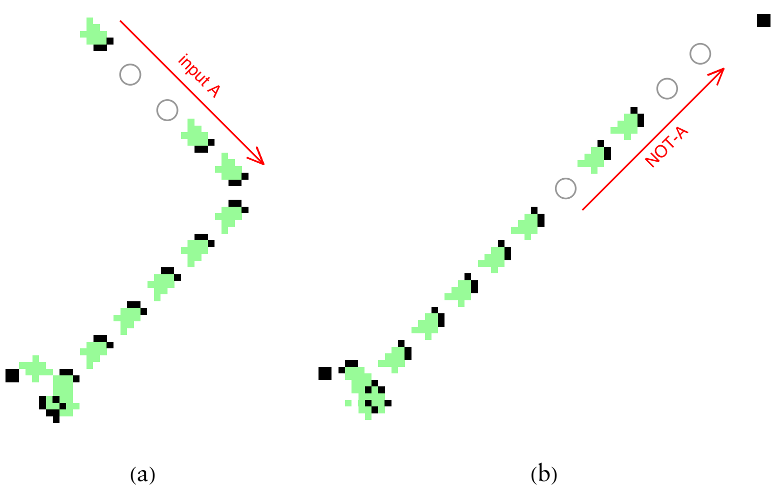
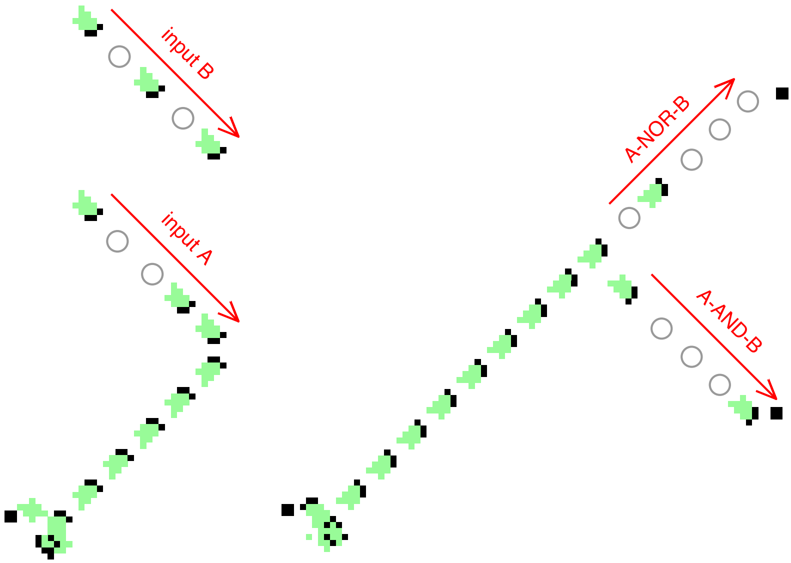


Figure 19. An example of the NOT gate: (¬ 1, 1 → 0 and 0 → 1) or inverter, which transforms a stream of data to its complement, represented by gliders and gaps. The five-bit input string A (11001) moving SE interacts with a GG1

glider stream moving NE, resulting in NOT-A (00110) moving NE, shown after 94 time steps.



(a) (b)

Figure 20. An example of the AND gate ($1 \wedge 1 \rightarrow 1$, else $\rightarrow 0$) making a conjunction between two streams of data, represented by gliders and gaps. The five-bit input strings A (11001) and B (10101) both moving SE interact with a GG1 glider stream moving NE, resulting in A-AND-B (10001) moving SE shown after 174 time steps. The dynamics making this AND gate first make an intermediate NOT-A string 00110 (as in Figure 19), which then interacts with input string B to simultaneously produce both the A-AND-B string moving SE described previously, and also the A-NOR-B string 00010 moving NE.

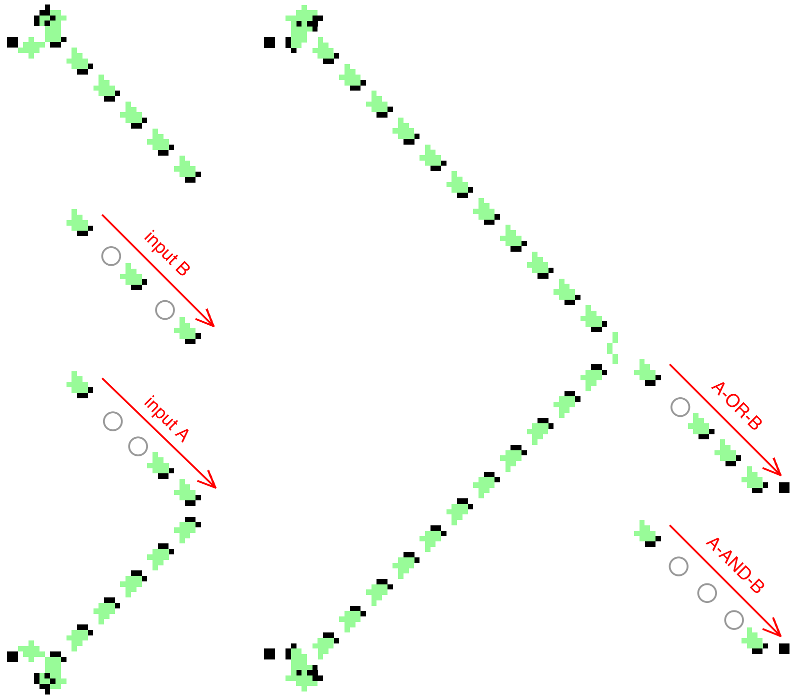


Figure 21. An example of the OR gate ($1 \vee 1 \rightarrow 1$, else $\rightarrow 0$) which makes a disjunction between two streams of data represented by two streams of gliders and gaps. The five-bit input strings A (11001) and B (10101) both moving SE interact with two GG1 glider streams, the lower GG1 shooting NE, and subsequently with an upper GG1 shooting SE, finally resulting in the A-OR-B string (11101) moving SE shown after 232 time steps. The dynamics first make an intermediate NOT-A string 00110 (as in Figure 19), which then interacts with string B to simultaneously produce both the AND string (10001, which appears in the figure) and an intermediate A-NOR-B string 00010—this is inverted by the upper glider gun stream to make NOT(A-NOR-B), which is the same as the A-OR-B string (11101).

5. Concluding Remarks

From the Sayab rule's glider gun and other artifacts, it is possible to build the logical gates NOT, AND and OR required for logical universality, which are constructed by collision dynamics depending on precise timing and points of impact. Furthermore, the fact that the glider gun can result from a collision between two gliders, or between a glider and a simple oscillator, opens up possibilities for making complex dynamical structures.

Three basic existential ingredients are proposed for constructing logical gates: a glider gun, an eater and self-destruction when two gliders collide at an angle. Rules with these ingredients are certainly elusive; in previous work [11, 12, 20] we described how they can nevertheless be found. These methods and the frequency of such rules in rule space require further research. The rules occur as families of genetically related rules—this aspect in itself requires investigation—for example, variants of the Sayab rule make up a family with related behavior.

Finally, the minimal size of the Sayab rule's glider gun is significant because it should make it easier to interpret its dynamical machinery, employing de Bruijn diagrams and other mathematical and computational tools. Such further research holds the promise of understanding how glider guns and related artifacts can exist, and so reveal the underlying principles of self-organization in cellular automata, and by extension in nature itself.

Acknowledgments

Experiments were done with Discrete Dynamics Lab [25, 26], Wolfram Mathematica and Golly. The Sayab rule was found during a collaboration at June workshops in 2017 at the DDLab Complex Systems Institute in Ariege, France, and also at the Autonomous University of Zacatecas, Mexico and in London, UK. J. M. Gómez Soto acknowledges his residency at the DDLab Complex Systems Institute and financial support from the Research Council of Zacatecas (COZCyT).

References

[1] J. von Neumann, *Theory of Self-Reproducing Automata* (edited and completed by A. W. Burks), Urbana, IL: University of Illinois Press, 1966.

[2] E. F. Codd, *Cellular Automata*, New York: Academic Press, 1968.

[3] E. R. Banks, *Cellular Automata*, AI Memo No. 198, MIT Artificial Intelligence Lab, Cambridge, MA, 1970. dspace.mit.edu/bitstream/handle/1721.1/5853/AIM-198.pdf?sequence=2.

[4] E. R. Banks, "Information Processing and Transmission in Cellular Automata," Ph.D. thesis, Department of Mechanical Engineering, MIT, Cambridge, MA, 1971.

[5] C. G. Langton, "Self-Reproduction in Cellular Automata," *Physica D: Nonlinear Phenomena*, **10**(1–2), 1984 pp. 135–144. doi:10.1016/0167-2789(84)90256-2.

[6] J. Byl, "Self-Reproduction in Small Cellular Automata," *Physica D: Nonlinear Phenomena*, **34**(1–2), 1989 pp. 295–299. doi:10.1016/0167-2789(89)90242-X.

[7] E. R. Berlekamp, J. H. Conway and R. K. Guy, "What Is Life?," *Winning Ways for Your Mathematical Plays*, Vol 2., New York: Academic Press, 1982 Chap. 25, pp. 817–850.

[8] M. Gardner, "Mathematical Games: The Fantastic Combinations of John Conway's New Solitaire Game 'Life,'" *Scientific American*, **223**(4), 1970 pp. 120–123.

[9] D. Eppstein, "Growth and Decay in Life-Like Cellular Automata," *Game of Life Cellular Automata* (A. Adamatzky, ed.), New York: Springer, 2010.

[10] E. Sapin, O. Bailleux, J.-J. Chabrier and P. Collet, "A New Universal Cellular Automaton Discovered by Evolutionary Algorithms," *Genetic and Evolutionary Computation (Gecco2004)* (K. Deb, ed.), Berlin, Heidelberg: Springer, 2004 pp. 175–187. doi:10.1007/978-3-540-24854-5_ 16.

[11] J. M. Gómez Soto and A. Wuensche, "The X-Rule: Universal Computation in a Nonisotropic Life-Like Cellular Automaton," *Journal of Cellular Automata*, **10**(3–4), 2015 pp. 261–294.

[12] J. M. Gómez Soto and A. Wuensche, "X-Rule's Precursor Is Also Logically Universal," *Journal of Cellular Automata*, **12**(6), 2017 pp. 445–473.

[13] A. Wuensche, "Glider Dynamics in 3-Value Hexagonal Cellular Automata: The Beehive Rule," *International Journal of Unconventional Computing*, **1**(4), 2005 pp. 375–398.

[14] A. Wuensche and A. Adamatzky, "On Spiral Glider-Guns in Hexagonal Cellular Automata: Activator-Inhibitor Paradigm," *International Journal of Modern Physics C*, **17**(7), 2006 pp. 1009–1026. doi:10.1142/S012918310600945X.

[15] M. Cook, "Universality in Elementary Cellular Automata," *Complex Systems*, **15**(1), 2004 pp. 1–40. www.complex-systems.com/pdf/15-1-1.pdf.

[16] S. Wolfram, *A New Kind of Science*, Champaign, IL: Wolfram Media, Inc., 2002.

[17] A. Wuensche. "Glider-Guns in 3D Cellular Automata." Discrete Dynamics Lab. (Jan 18, 2018) uncomp.uwe.ac.uk/wuensche/multi_value/3d_glider _guns.html.

[18] P. Rendell, "Turing Universality of the Game of Life," *Collision-Based Computing* (A. Adamatzky, ed.), London: Springer, 2002 pp. 513–539. doi:10.1007/978-1-4471-0129-1_ 18.

[19] A. Wuensche and M. J. Lesser, "The Global Dynamics of Cellular Automata," *Santa Fe Institute Studies in the Sciences of Complexity*, Reading, MA: Addison-Wesley, 1992.

[20] A. Wuensche, "Classifying Cellular Automata Automatically: Finding Gliders, Filtering, and Relating Space-Time Patterns, Attractor Basins, and the Z Parameter," *Complexity*, **4**(3), 1999 pp. 47–66.

[21] C. G. Langton, "Computation at the Edge of Chaos: Phase Transitions and Emergent Computation," *Physica D: Nonlinear Phenomena*, **42**(1–3), 1990 pp. 12–37. doi:10.1016/0167-2789(90)90064-V.

[22] E. Sapin, A. Adamatzky, P. Collet and L. Bull, "Stochastic Automated Search Methods in Cellular Automata: The Discovery of Tens of Thousands of Glider Guns," *Natural Computing*, **9**(3), 2010 pp. 513–543. doi:10.1007/s11047-009-9109-0.

[23] E. L. Post, "The Two-Valued Iterative Systems of Mathematical Logic," *Annals of Mathematics Series 5*, Princeton, NJ: Princeton University Press, 1941. www.jstor.org/stable/j.ctt1bgzb1r.

[24] N. M. Martin and F. J. Pelletier, "Post's Functional Completeness Theorem," *Notre Dame Journal of Formal Logic*, **31**(3), 1990 pp. 462–475. doi:10.1305/ndjfl/1093635508.

[25] A. Wuensche, *Exploring Discrete Dynamics*, 2nd ed., Frome, UK: Luniver Press, 2016.

[26] A. Wuensche. Discrete Dynamics Lab (DDLab). (Jan 18, 2018) www.ddlab.org.