Black Hole Tech?

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The Theory Works!

The equation that Albert Einstein wrote down for the gravitational field in 1915 is simple enough:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

But working out its consequences is not. And in fact even after 100 years we're still just at the beginning of the process.

```
In[1]:= DefManifold[M, 4, {α, β, γ, δ, μ, ν}]

In[2]:= DefMetric[-1, g[-μ, -ν], cd]

In[3]:= DefMetricPerturbation[g, h, ϵ]

In[4]:= Perturbed[Riemanncd[-α, -β, -γ, δ], 3]
```

Out[4]=

$$\epsilon \triangle\left[R_{\alpha\beta\gamma}{}^{\delta}\right] + \frac{1}{2}\epsilon^2 \triangle^2\left[R_{\alpha\beta\gamma}{}^{\delta}\right] + \frac{1}{6}\epsilon^3 \triangle^3\left[R_{\alpha\beta\gamma}{}^{\delta}\right] + R_{\alpha\beta\gamma}{}^{\delta}$$

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In[5]:= Collect[ExpandPerturbation[%], ϵ, Simplification]
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Out[5]=

$$R_{\alpha\beta\gamma}{}^{\delta} + \frac{1}{2}\epsilon\Big(- h^1{}_{\gamma}{}^{\delta}{}_{;\beta;\alpha} - h^1{}_{\beta}{}^{\delta}{}_{;\gamma;\alpha} +$$
$$h^1{}_{\beta\gamma}{}^{;\delta}{}_{;\alpha} + h^1{}_{\gamma}{}^{\delta}{}_{;\alpha;\beta} + h^1{}_{\alpha}{}^{\delta}{}_{;\gamma;\beta} - h^1{}_{\alpha\gamma}{}^{;\delta}{}_{;\beta}\Big) +$$
$$\frac{1}{4}\epsilon^2\Big(- h^2{}_{\gamma}{}^{\delta}{}_{;\beta;\alpha} - h^2{}_{\beta}{}^{\delta}{}_{;\gamma;\alpha} + h^2{}_{\beta\gamma}{}^{;\delta}{}_{;\alpha} + h^{1\delta}{}_{\mu;\alpha}\, h^1{}_{\gamma}{}^{\mu}{}_{;\beta} -$$
$$h^1{}_{\gamma}{}^{\mu}{}_{;\alpha}\, h^{1\delta}{}_{\mu;\beta} + h^2{}_{\gamma}{}^{\delta}{}_{;\alpha;\beta} + h^2{}_{\alpha}{}^{\delta}{}_{;\gamma;\beta} - h^2{}_{\alpha\gamma}{}^{;\delta}{}_{;\beta} +$$
$$2\, h^{1\delta\mu}\Big(h^1{}_{\gamma\mu;\beta;\alpha} + h^1{}_{\beta\mu;\gamma;\alpha} - h^1{}_{\beta\gamma;\mu;\alpha} - h^1{}_{\gamma\mu;\alpha;\beta} -$$
$$h^1{}_{\alpha\mu;\gamma;\beta} + h^1{}_{\alpha\gamma;\mu;\beta}\Big) - h^{1\delta}{}_{\mu;\beta}\, h^1{}_{\alpha}{}^{\mu}{}_{;\gamma} +$$
$$h^{1\delta}{}_{\mu;\alpha}\, h^1{}_{\beta}{}^{\mu}{}_{;\gamma} + h^1{}_{\gamma\mu;\beta}\, h^1{}_{\alpha}{}^{\mu;\delta} + h^1{}_{\beta\mu;\gamma}\, h^1{}_{\alpha}{}^{\mu;\delta} -$$
$$h^1{}_{\alpha}{}^{\mu}{}_{;\gamma}\, h^1{}_{\beta\mu}{}^{;\delta} - h^1{}_{\gamma\mu;\alpha}\, h^1{}_{\beta}{}^{\mu;\delta} - h^1{}_{\alpha}{}^{\mu;\delta}\, h^1{}_{\beta\gamma;\mu} +$$
$$h^1{}_{\alpha}{}^{\mu}{}_{;\gamma}\, h^1{}_{\beta}{}^{\delta}{}_{;\mu} + h^{1\delta}{}_{\mu;\beta}\, h^1{}_{\alpha\gamma}{}^{;\mu} + h^1{}_{\beta\mu}{}^{;\delta}\, h^1{}_{\alpha\gamma}{}^{;\mu} -$$
$$h^1{}_{\beta}{}^{\delta}{}_{;\mu}\, h^1{}_{\alpha\gamma}{}^{;\mu} - h^1{}_{\gamma\mu;\beta}\, h^1{}_{\alpha}{}^{\delta;\mu} - h^1{}_{\beta\mu;\gamma}\, h^1{}_{\alpha}{}^{\delta;\mu} +$$

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Millions of lines of algebra have been done along the way (often courtesy of Mathematica and the Wolfram Language). And there have been all sorts of predictions. Like that if two black holes merge, there should be a burst of gravitational radiation generated, with a particular form. And a little more than a week ago—in a triumph of theoretical and experimental science—it was announced that just such gravitational radiation had been detected.

I've followed General Relativity and gravitation theory for more than 40 years now—and it's been inspiring to see how the small community that's pursued it has progressively increased its theoretical prowess, and how the discussions I saw at Caltech in the late 1970s finally led to a successful detector of gravitational waves.

General Relativity is surely not the whole story of how spacetime and gravity work. But we've now just got some spectacular new evidence of how far the theory can be taken. For a long time I myself was a bit skeptical about black holes—and for example about whether true General-Relativity-style ones would actually form in real physical processes. But as of a little more than a week ago I'm finally convinced that black holes exist, just as General Relativity suggests.

Fast Forward to Black Hole Technology

OK, so we've observed one pair of black holes, a billion light years away. And no doubt now—quite amazingly—we'll get evidence for a steady stream of others around the universe. But what if somehow we could get our hands on our very own black holes, and maybe even lots of them? What could we—or, for that matter, any putative extraterrestrials—do with them? What kind of perhaps extremely exotic structures or technology could eventually be made with them?

It's always the same story with technology. We have to take the raw material that our universe provides, and somehow find ways to organize it for purposes we want. It's remarkable to look through the list of chemical elements, or a list of physics effects that have been discovered, and to realize that—though it sometimes took a while—almost all those that can be readily realized on the time and energy scales of today's technology have found real applications. So what about black holes? Given how hard it's been to detect our very first pair of black holes, it might seem almost irreverent to ask. And perhaps our universe just isn't big enough for the question to be sensible. But as a kind of celebration of the detection of gravitational waves I thought it might be fun to try fast-forwarding a long way—and seeing what one can figure out about technology that black holes could make possible.

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It seems inconceivable that we ourselves will ever get to try out anything like this for real—unless we find a way to locally make tiny stable black holes. But if something is possible to do, perhaps some more-advanced civilization out there in the universe has already done it—but we likely couldn't recognize evidence of it without having more idea of what's possible.

But before we can get to speculating about black hole technology, we're going to have to talk a bit about what's known about black holes, General Relativity and gravitation. There are lots of complicated issues—that are probably most easily explained using some fairly mathematically sophisticated concepts (Riemann tensors, covariant derivatives, spacelike hypersurfaces, Penrose diagrams, etc. etc.). But for the sake of writing a general blog post, I'm going to try to do without these, while still, I hope, correctly communicating what's known and what's not. I won't be able to do it perfectly, and might lapse unwittingly into physics-speak from time to time, but here goes…

What Are Black Holes?

General Relativity is often discussed in terms of the geometry of spacetime. But one can also think of it as just saying that gravity is associated with a field that has a certain strength or value at every point. This idea of a field is basically just like in electromagnetism, with its electric and magnetic fields. It's also like in fluid mechanics, where there's a velocity field that gives the velocity of the fluid at every point (like a wind velocity map for the weather).

What Einstein did in 1915 was to suggest particular equations that should be satisfied by the gravitational field. Mathematically, they're partial differential equations, which means that they say how values of the field relate to rates of change (partial derivatives) of these values. They're the same general kind of equations that we know work for electromagnetic fields, or for the velocity field in a fluid.

So what does one do with these equations? Well, one solves them to find out what the field is in any particular case. It turns out that for electromagnetism, the structure of the equations makes this in principle straightforward. But for fluid mechanics, it's considerably more complicated—and for Einstein's equations it's much more complicated still.

In electromagnetism, one can just think of charges and currents as being sources of electromagnetic field, and there's no "internal effect" of the field on itself (unless one considers quantum effects). But for fluid mechanics and Einstein's equations, it's a different story. In a first approximation, the velocity of a fluid is determined by whatever pressure is applied to it. But what complicates things greatly is that within the fluid there's an internal effect of each part of the velocity field on others. And it's similar with the gravitational field: In a first approximation, the field is just determined by whatever configuration of masses exists. But there's also an "internal effect" of the field on itself. Physically, this is because the gravitational field can be thought of as having energy and momentum, which behave like mass in effectively being a source of the field. (The electromagnetic field has energy and momentum too, but it doesn't itself have charge, so doesn't act as a source for itself. In QCD, the color field itself has color, so it has the same general kind of nonlinear character as fluid mechanics or Einstein's equations.)

In electromagnetism, with its simpler structure, one can't have any region of static nonzero field unless one has charges or currents explicitly producing it. But when fields can act on themselves it's a different story, and there can be structures that exist purely in the field, without any external sources being present. For example, in a fluid there can be a vortex that just exists within the fluid—because this happens to be a possible solution to the pure equations for the velocity field of the fluid, without any external forces.

What about the Einstein equations? Well, it's somewhat the same story, though the details are considerably more complicated. There are nontrivial solutions to the Einstein equations even in the case of "pure gravity", without any matter or external configuration of masses being present. And that's exactly what black holes are. They're examples of solutions to the Einstein equations that correspond to structures that can just exist independently in a gravitational field, a bit like vortices can just exist in the velocity field of a fluid.

How Black Holes Are Made

From everyday experience and from seeing the operation of programs, we tend to be used to the idea that the way to work out what something will do is to start from the beginning and then go forwards step by step. But in mathematically based science the setup is often much less direct and constructive, and instead is basically "the system obeys such-and-such an equation; whatever the system does must correspond to some solution or another to the equation". And that's ultimately the setup with Einstein's equations.

There can be some serious complications. For example, given particular constraints it's far from obvious that any solutions to the equations will exist, or be unique. And indeed we'll encounter difficulties along these lines later. But let's start off by trying to get some rough idea of the physics of how black holes can be made.

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The classic way one imagines a black hole is made is from the collapse of a massive star. And that's presumably where the two black holes just detected came from.

For the Earth, with its particular mass and radius, we can work out that something launched from the surface must have a velocity of about 25,000 miles per hour to escape Earth's gravity. But for a body whose mass is larger or whose radius is smaller, the escape velocity will be larger. And what General Relativity (like Newtonian gravity before it) says is that eventually the escape velocity will exceed the speed of light—so that neither light nor anything else will be able to escape, so the object will always seem black: a black hole.

When this happens, there's inevitably also a strong gravitational field. And this gravitational field effectively has mass, which itself serves as a source of gravitational field. And in the end, it's actually irrelevant if there's matter there at all: the black hole is in effect a self-sustaining configuration of the gravitational field that exists as a solution to Einstein's equations. It's a bit like a vortex in a fluid, which you can start by stirring, but which, once it's there, effectively just perpetuates itself (though in a real fluid with viscosity it'll eventually damp out).

It's not obvious of course that the mass and radius needed to get a black hole would actually occur. It's known that stars like the Sun will never collapse far enough. But above about 3 or 4 solar masses, there's at least no known physical process that will prevent a star from collapsing enough to form a black hole. And the 36- and 29-solar-mass black holes recently observed presumably formed this way.

What Can a Black Hole Be Like?

Let's for a moment ignore how black holes might be formed, and just ask what they can be like. This is really a question about possible solutions to Einstein's equations. And if we want something that doesn't change with time, and that's localized in space, then there are mathematical theorems that say the choices are very limited.

There could have been a whole zoo of possible black hole structures—and in higher dimensions, there are at least a few more. But for 4D spacetime, it actually turns out that all stationary black hole solutions are mathematically similar, and are determined by just two parameters: their overall mass and angular momentum. (If one includes electromagnetism as well, then they're also determined by charge—and it'd be the same story with any other long-range gauge fields.)

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The case of non-rotating black holes (zero angular momentum) is simplest. The relevant solution to the Einstein equations was found already by Karl Schwarzschild in 1915. But it took nearly 50 years for the interpretation of the solution to become clear.

One crucial feature of the Schwarzschild solution is that it has an event horizon. This means that any light rays (or anything else) that originate inside a certain sphere (the event horizon) are trapped forever, and can't escape. There was confusion for quite a while, because the original formula for the Schwarzschild solution has a singularity at the event horizon. But actually this is just a mathematical artifact that can be removed by using a different coordinate system, and isn't relevant to anything physically observable.

But even though there's no real singularity at the event horizon, there is a singularity at the very center of the black hole—where the curvature of spacetime, and thus the effective strength of the gravitational field, is infinite. And it turns out that this singularity is in effect where the whole mass of the black hole is concentrated. It's a pretty pathological situation. If this were happening in fluid mechanics, for example, we'd just assume that the continuum differential equations we're using must break down, and that instead we'd have to work at the level of molecules. But for General Relativity we don't yet have any established lower-level theory to use (though I certainly have ideas, and string theory has claims of being able to come to the rescue). There's also elegant mathematics that's developed around black holes and their singularities—and anyway at least in this case one can say that "It's all happening inside the event horizon so nobody outside will ever find out about it". So the current state of the art is just to work with the theory assuming the singularity is real—and what's interesting now is that calculations based on this seem to have given correct answers for the recent gravitational wave discovery.

The Life of a Non-rotating Black Hole

I just talked a bit about the mathematical structure of a black hole solution to Einstein's equations. But how does this correspond to an actual black hole that could form from the collapse of a massive star?

The truest way to find out would be to start from an accurate model of the star and then simulate the whole process of forming the black hole. And at least in some approximation, it's possible these days to do this. But let's try a more lightweight approach.

Let's assume that there's a black hole solution to Einstein's equations that exists. Then let's ask what happens when small things fall into it. Well, there's already an issue here. Think about an observer far from the black hole. In order to "get the news" that something crossed the event horizon of the black hole, the observer would have to get some signal—say a light pulse. But as the thing gets closer to the event horizon, it'll take longer and longer for the signal to escape. And the result is that the observer will never see things cross the event horizon: they'll appear to get closer and closer (and darker and darker), but never actually cross.

And that'll be true even when it comes to the formation of the black hole. The star will be seen to be collapsing, but it'll look as if it's just freezing when it gets to the point where an event horizon would form.

OK, but what if the observer is also falling into a black hole? Here the experience is completely different. They probably wouldn't even notice when they cross the event horizon, except that "handshake" signals to the outside world will stop getting responses. But then they'll get pulled in towards the singularity at the center of the black hole. The gravitational field will steadily increase, and the fact that it's stronger further in will inevitably stretch any object (or observer!) out. But eventually, splat, they'll hit the singularity—and in some sense be sucked into it.

Is that really how things will work? Well, it's hard to tell, but probably not. Outside the event horizon it's known that small perturbations in the structure of the gravitational field—say associated with the presence of matter—will tend to get damped out, so that what emerges is exactly the official Schwarzschild black hole solution to the Einstein equations.

But inside the event horizon it's much less clear what happens. As soon as there are perturbations, there'll be time variations in the gravitational field, and one's no longer dealing with a static solution to the Einstein equations. The result is that the known theorems no longer apply—and quite possibly there'll be instabilities that change the structure or even existence of the singularity. But at least in this case, in some sense it doesn't matter—because none of what happens will ever be visible outside of the event horizon.

Rotating Black Holes

In 1963 Roy Kerr found a solution to Einstein's equations that corresponds to a black hole with angular momentum. Like the solution for a non-rotating black hole, it has a singularity in the middle. But now the singularity is not a point; instead it forms a ring.

And at least so long as the angular momentum J is (in suitable units) less than the square of the mass, M^2, the rotating black hole solution has an event horizon. And outside the event horizon, pertur-

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bations tend to get damped, just like in the non-rotating case. But inside, things are different.

In a non-rotating black hole anything that goes inside the event horizon will eventually hit the singularity, but won't "see it coming". And if light or anything else originates at the singularity it'll just stay there, and never "get out".

But the same isn't true in a rotating black hole. Here, not everything will hit the singularity, and things that originate at the singularity can "get out". This latter point is quite a problem—because it means that to know the behavior inside the black hole, you have to know what happens at the singularity. But at the singularity, Einstein's equations can't tell one anything: they essentially just say infinity=infinity. So the conclusion is that at least based on Einstein's equations, one simply can't predict what will happen.

At least with $J < M^2$, this failure of prediction occurs only inside the so-called inner horizon of the black hole. But even outside this, something weird happens. To an observer falling into the black hole, it'll seem like a finite time elapses between when they cross the event horizon and the inner horizon. But to an observer outside the black hole, this will seem like an infinite time. And that means that any signals that come from outside the black hole—into the infinite future—could be collected by the observer inside the black hole, in finite time.

Most likely this is a sign that in practice unbounded amounts of energy will accumulate near the inner horizon, making it unstable. But if somehow stability were maintained, there'd be a really weird effect going on: the observer inside the black hole would get to see, in finite time, the whole infinite future unfolding outside the black hole. And if that future happened to include Turing machines doing computations, then in finite time the observer would get to see computations—like solving the halting problem—that can't necessarily be done by Turing machines in any finite time.

This might be billed as evidence for "physics going beyond the Turing limit", but it's not really convincing, first because the whole theoretical internal structure of rotating black holes probably gets modified in practice; and second, because to really talk about the infinite future we have to consider the structure of the whole universe, not just one specific black hole.

But despite all this complexity about what happens inside the event horizon, General Relativity has clear predictions for outside—and these are what were needed for the pair of black holes just detected.

Naked Singularities

In a rotating black hole with $J < M^2$, there's a nasty singularity—but it's safely inside an event horizon. But for $J > M^2$, there's the same kind of singularity, but now it's no longer inside an event horizon, and instead it's "naked" and exposed to the outside universe.

If there's a naked singularity like this, the consequence is simple: General Relativity alone isn't sufficient to describe what happens in the universe; some additional theory is needed.

Encountering something like this is one of the hazards of using a theory—like General Relativity—that's based on solving equations (rather than, say, running a program) to deduce how systems behave.

And in fact, it's still quite possible that something similar happens in the Navier–Stokes equations for fluid mechanics. There are lots of partial results, but it's still not known whether starting from smooth initial conditions, the Navier–Stokes equations can generate singularities.

From a physics point of view, though, there's something to say: the Navier–Stokes equations for fluids are derived by assuming that the velocity field doesn't change too rapidly in space or time. And that's a fine assumption when the velocities are small. But as soon as there's supersonic flow, there are shocks where the velocity changes rapidly. Viscosity smooths out the shocks a bit, but by the time one's in the hypersonic regime, at Mach 4 or so, the shocks get very sharp—in fact, so sharp that their width is less than the typical distance between collisions for molecules in the fluid. And the result of this is that the continuum description of the fluid necessarily breaks down, and one has to start looking at the underlying molecular structure.

OK, so can naked singularities actually occur in practice in General Relativity? We know they occur if you somehow have a $J > M^2$ object. But what if you start from a realistic star, or some other distribution of matter? Can it spontaneously evolve to produce a naked singularity?

It was proved a few decades ago that if you start with something that's close to ordinary flat spacetime, it can't spontaneously make singularities. But if you start putting matter in, then the story changes. And in fact there are now several examples known where a smooth initial distribution of matter can evolve to make a naked singularity—though the singularity only shows up if the initial conditions are very carefully arranged and as soon as there's any perturbation, it goes away.

Can one get a stable naked singularity without this kind of special setup? So far, nobody knows.

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And nobody knows whether $J > M^2$ objects can be formed. If one looks at candidate black holes around the universe, most of them are rotating. The final one from the week before last had $J \simeq 0.7M^2$. And it's certainly interesting to note that while many have J close to M^2, none seen so far have $J > M^2$. It's also interesting that in numerical simulations of pairs of rotating black holes, they always eventually merge—but if the result would have $J > M^2$ they seem to "delay" their merger, and emit lots of gravitational radiation that gets rid of angular momentum, before merging to produce a black hole with $J < M^2$.

Gravitational Waves

People have been talking about gravitational waves for almost a century, and there's been indirect evidence of them for a while. But the recent announcement of direct detection of gravitational waves is pretty exciting.

So what are gravitational waves? They're a fairly direct analog of electromagnetic waves. If you take a charge and wiggle it around, it'll radiate electromagnetic waves—for example, radio waves. And in a directly analogous way, if you take a mass and wiggle it around, it'll radiate gravitational waves. Usually they'll be incredibly weak. But if the mass is very big and concentrated, like a black hole, the gravitational waves can be stronger—and, as we've now seen, even strong enough to detect.

Why is there radiation when you wiggle something around? It's not hard to see. Imagine, say, that there's a charge sitting somewhere, and you're some distance away. There'll be electric field from the charge—that's, say, pointing towards the charge. Now suddenly move the charge. After things have stabilized again, there'd better be a new version of the electric field, say pointing to the new position of the charge. But how does the transition happen? The answer is that the change somehow has to propagate outward from the charge—and the process of that happening is electromagnetic radiation, which (in a vacuum) moves at the speed of light.

In general, the amount of electromagnetic radiation that's produced is proportional to (the square of) the acceleration of the charge. (Actually, there's considerable subtlety to this, particularly in the relativistic case—and the details of the globally correct formula are still somewhat debated.) It's similar for gravitational radiation.

There are some differences though. A minimal antenna for electromagnetic radiation is a straight wire, that electrons can go up and down. For gravitational radiation, the minimal "antenna" has to be

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something that effectively has motion in two perpendicular directions—or, more technically, a changing quadrupole moment. In practice, two bodies orbiting each other will emit gravitational radiation, more or less as a result of the acceleration necessary to keep them in their orbits. More or less any mass that "blobs around" without being spherically symmetric will also emit gravitational waves.

When something emits gravitational waves, it's radiating away some of its energy. And in general the emission of gravitational radiation tends to have a damping effect on the motion of things. For example, the emission of gravitational radiation will make orbits decay—and makes orbiting bodies progressively spiral in towards each other.

For something like the Earth and the Sun, this is an absolutely infinitesimal effect. But for a pair of neutron stars orbiting each other, it's more significant. And indeed, starting in 1974 such an effect was observed in a binary pulsar. And now, this is what caused two black holes eventually to spiral in so far that they hit each other—and produce the event just announced.

Once two black holes hit, there's a tremendous amount of gravitational radiation emitted as the resulting object "blobs around" before assuming its final single-black-hole shape. For stellar-sized black holes it all happens in a few hundred milliseconds. And in the case of the event just announced, the total energy in gravitational radiation was a whopping 3 solar masses—big enough that we're able to detect it a significant fraction of the way across the universe.

Mathematics of Waves

Pretty much any kind of field or continuous material supports some kind of waves. Start from whatever the stable state of the system is, then perturb it just a little by periodically changing something, and you'll get waves. When the amplitude of the waves is small enough, the math tends to be fairly straightforward. For example, in a first approximation, the amplitudes of different waves at a particular point will just add linearly.

But when the amplitudes of the waves get bigger, things can get much more complicated. In electromagnetism, everything stays linear however big the amplitude is (well, until one runs into quantum effects). But for pretty much any other kind of waves—including, say, water waves, as well as gravitational waves—there start to be nonlinear effects as soon as the amplitude is larger.

When there's linearity, one can effectively break down any field configuration into a sequence of non-interacting waves of different frequencies. But that's no longer true for something nonlinear, and

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eventually it usually doesn't make sense to talk about waves at all: one's just dealing with some field configuration or another.

In the case of gravitational waves, one of the notable features is that one can in principle arrange waves to combine so that they'll form black holes. Indeed, one can potentially start with low-amplitude waves, but somehow make them converge to a point where they'll generate a black hole (think "gravitational implosion lens", etc.).

Two Black Holes

A single static black hole in an infinite universe is a possible solution to Einstein's equations. So what about two black holes orbiting each other? Well, there's no known exact solution to the equations for this case, and it's only fairly recently that it's become possible to calculate with any reliability what happens.

Roughly, there are three regimes. First, the black holes are peacefully orbiting, and emitting gravitational radiation. When the black holes are far apart, and have velocities small compared to the speed of light, it's fairly straightforward. But as they get closer and speed up, it becomes more complicated. Each black hole perturbs the other, but with a lot of algebra it's possible to calculate the effects (as a power series in v/c).

Eventually, though, this breaks down, and the only choice is to solve the Einstein equations numerically using many of the same methods traditionally used for fluid mechanics. (There've been various efforts to use the same kind of cellular automaton approach on the Einstein equations that I used for the Navier–Stokes equations, but I think what's more promising is to try something like my network-rewriting models for gravity.)

It's only in recent years that computers have become fast enough to get sensible answers from computations like this involving high gravitational fields as well as velocities close to the speed of light. And in these computations, the result is that something like a single black hole is formed. Inevitably it's a deformed black hole, and the third regime is one where—a bit like a bell—the black hole "rings down" these deformations (either by emitting gravitational radiation, or by absorbing them into the black hole itself).

It's a pretty complicated stack of computations, requiring a variety of different methods. But the impressive thing is that—judging from the recent announcement—it seems to correctly capture what goes on in the interaction between two black holes.

There are plenty of detailed issues, however. One of them is that you can't just set up some elaborate initial state with two black holes and expect that it will be a solution to the Einstein equations, even for an instant. So in addition to working out the time evolution, one also has to somehow progressively modify the initial conditions one specifies, so that they actually correspond to a possible configuration of the gravitational field according to Einstein's equations.

If we want to start thinking about black hole configurations for purposes of technology, it would help to devise a simplified summary of interactions between two—or more—black holes. For example, one might want to have a summary of the effects of the direction of rotation (or "spin") and of orbiting on black holes' interactions, organized (in analogy with quantum systems) into spin-orbit, spin-spin, etc. components.

Gravitational Turbulence?

It's a general feature of fluids that when they flow rapidly, they tend to show turbulence and behave in seemingly random ways. It's still not completely clear what the origin of this apparent randomness is. It could be that somehow one is seeing an amplified version of small-scale random molecular motions. Or it could be there is enough instability that one is progressively exploring random details of initial conditions (as in chaos theory). I've spent a long time studying this, and my conclusion is that the randomness mostly isn't coming from things that are essentially outside of the fluid; it's instead coming from the actual dynamics of the fluid, as if the fluid were computing my rule 30 cellular automaton, or running a pseudorandom number generator.

If one works with the standard Navier–Stokes equations for fluid mechanics, it's not very clear what's going on—because one ends up having to solve the equations numerically, and whenever something complicated happens, it's almost impossible to tell if it's a consequence of the numerical analysis one's done, or a genuine feature of the equations. I sidestepped these issues by using cellular automaton models for fluids rather than differential equations—and from that it's pretty clear that intrinsic randomness generation is at least a large part of what's going on. And having seen this, my expectation would be that if one could solve the equations well enough, one would see exactly the same behavior in the Navier–Stokes equations.

So what about the Einstein equations? Can they show turbulence? I've long thought that they should be able to, although to establish this will run into the same kinds of numerical-analysis issues as with

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the Navier–Stokes equations, though probably in an even more difficult form.

In a fluid the typical pattern is that one starts with a large-scale motion (say induced by an airplane going through the air). Then what roughly happens (at least in 3D) is that this motion breaks down into a cascade of smaller and smaller eddies, until the eddies are so small that they are damped out by viscosity in the fluid.

Would something similar happen with turbulence in the gravitational field? It can't be quite the same, because unlike fluids, which dissipate small-scale motion by turning it into heat, the gravitational field has no such dissipation mechanism, at least according to Einstein's equations (without adding matter, quantum effects, etc.). (Note that even with ordinary fluid mechanics, things are very different in 2D: there eddies tend not to break into smaller ones, but instead to combine into larger ones, perhaps like the Great Red Spot on Jupiter.)

My guess is that a phenomenon akin to turbulence is endemic in systems that have fields which can interact with themselves. Another potential example is the classical analog of QCD—or, more simply, classical Yang–Mills theory (the theory of a classical self-interacting color field). Yang–Mills theory shares with gravity the feature that it exhibits no dissipation, but is mathematically perhaps simpler. For years I've been asking people who do lattice-gauge-theory simulations whether they see any analog of turbulence. But with the randomized sampling (as opposed to evolution) approach they typically use, it's hard to tell. (There are mathematical connections between versions of gravity and versions of Yang–Mills theory that have been extensively explored in recent years, but I don't know what implications they have for questions of turbulence.)

Orbiting a Black Hole

In Newton's theory of gravity, there's an inverse square law for the force of gravity. Sufficiently far away from a massive object, the same law holds in General Relativity too. With an inverse square law for gravity, the orbit of a pointlike object around any spherical mass will always be an ellipse (just like Newton said it should be for Halley's Comet). And every time the object goes around its orbit, it will just retrace the exact same ellipse, keeping the long axis of the ellipse in the same direction.

But what happens in General Relativity, and with black holes? The first important fact is that if something is spherically symmetric, then the gravitational field it produces outside itself must always be given

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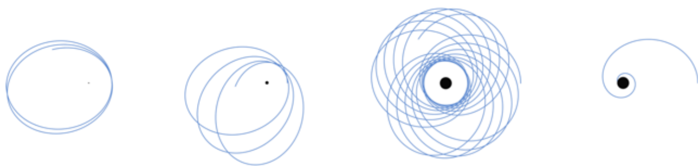
exactly by the Schwarzschild solution to Einstein's equations. That's true for a perfectly spherical star, and it's also true for a non-rotating black hole. And in fact that's why it was often hard to tell if you were dealing with a genuine black hole: because the gravitational field outside it would be the same as for a star of the same mass.

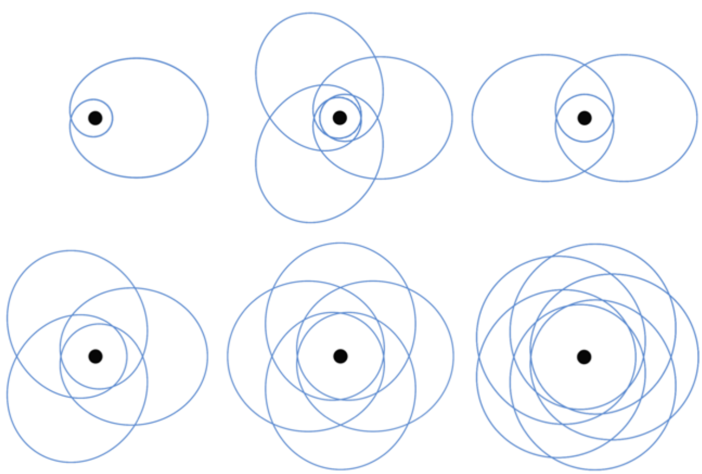
So what happens according to General Relativity if you're in orbit around something spherical? In a first approximation, the orbit is still elliptical, but the axis of the ellipse can change ("precess")—and in fact one of the early successes of General Relativity was to explain an effect like this that had been seen for the orbit of the planet Mercury (the "advance of the perihelion").

Here's what actually happens as the orbital distance goes down:

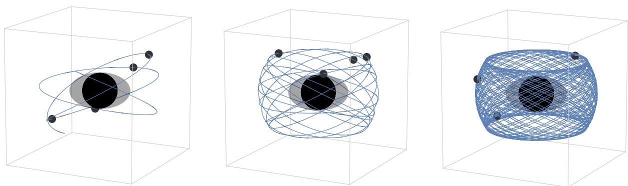


The object in the middle looks larger and larger relative to the orbit. In the final picture, there's no orbit at all, and one just spirals into the object in the middle. In the other cases, there are roughly elliptical orbits, but the precession effect gets larger and larger, and typically one ends up eventually visiting a whole ring of possible positions. (There's an interactive version of this on the Wolfram Demonstrations Project.)

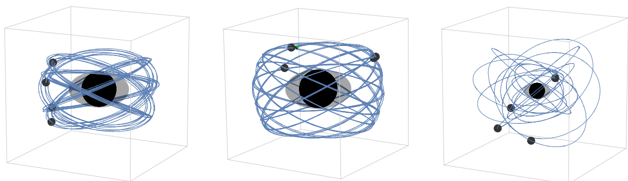
But does this always happen? The answer is no: one can pick special initial conditions that instead give a variety of closed orbits with various patterns:



So what about a rotating object, or specifically a rotating black hole? One notable feature is a phenomenon called "frame dragging", which causes orbits to be pulled towards rotating along with the object. A consequence of this is that unless the orbit precisely follows the direction of rotation, it won't stay in a single plane, and—in a seemingly quite random way—will typically fill up not a ring but a whole 3D torus. (Try out the interactive demonstration to see this.)



Although it eventually fills in a torus, the pattern of the orbit can be fairly different depending on what initial "latitude" one starts from (all these are shown for the same total time):



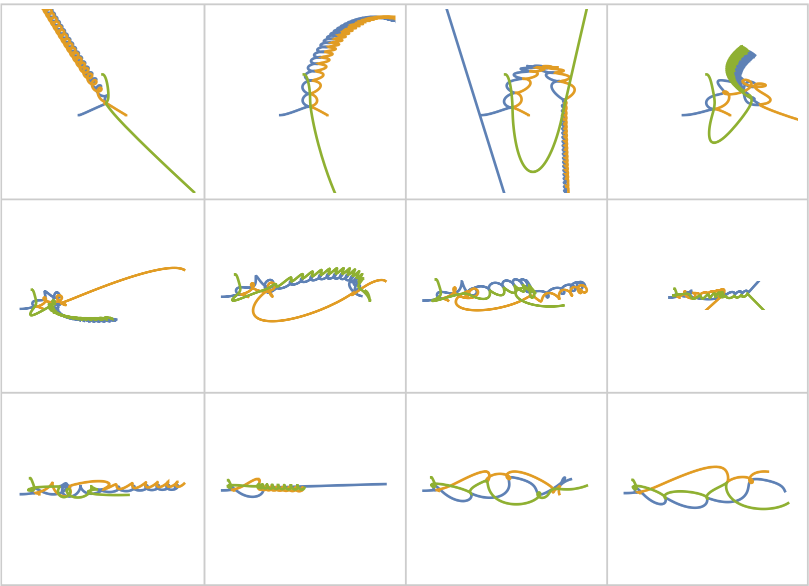
If you're sufficiently far away from the black hole, then it turns out that even though you're pulled by frame dragging, you can in principle overcome the force (say with a powerful enough rocket). But if you're inside a region called the ergosphere (indicated by the gray region in the pictures), you'd have to be going faster than the speed of light to do that. So the result is that any object that gets into the ergosphere (which extends outside of the event horizon) will inevitably be made to co-rotate with the black hole, just through frame dragging.

And this means that if you can put something into the ergosphere, it can gain energy—ultimately by reducing the angular momentum of the black hole. One could imagine using this as a way to harvest the energy of a black hole—and indeed astronomical phenomena like high-energy gamma ray bursts are thought to be possibly related.

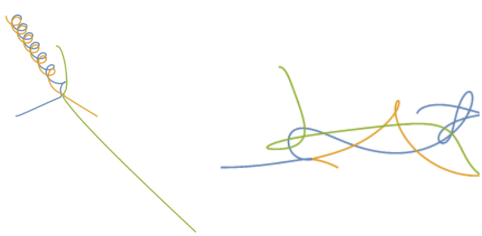
The Three-Body Problem

OK, so we've talked about orbiting a black hole, and earlier about what happens with two black holes. But what about with more black holes? Well, we can start by asking that question just for simple point masses following Newton's law of gravity—and it turns out that even there things are already extremely complicated.

The pictures below show a bunch of possible trajectories for three equal-mass pointlike objects interacting through ordinary Newtonian gravity. The only difference between the setup for the different pictures is where the objects were started. But one can see that just changing this initial condition leads to an incredible diversity of behavior:



Here are some animated versions:



Solving the necessary differential equations is fast enough these days in the Wolfram Language that one can actually generate these interactively. Here's a version in 2D where you can interactively move around the initial positions and velocities:

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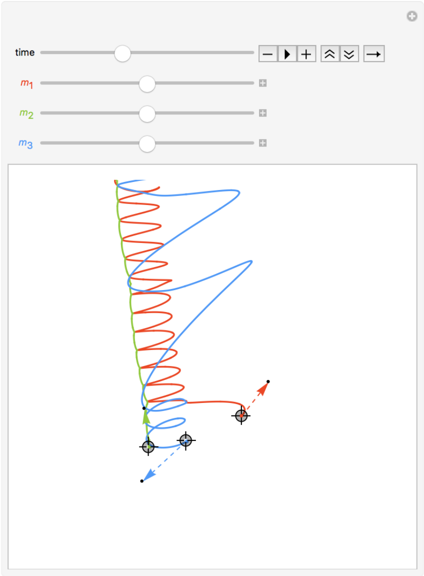
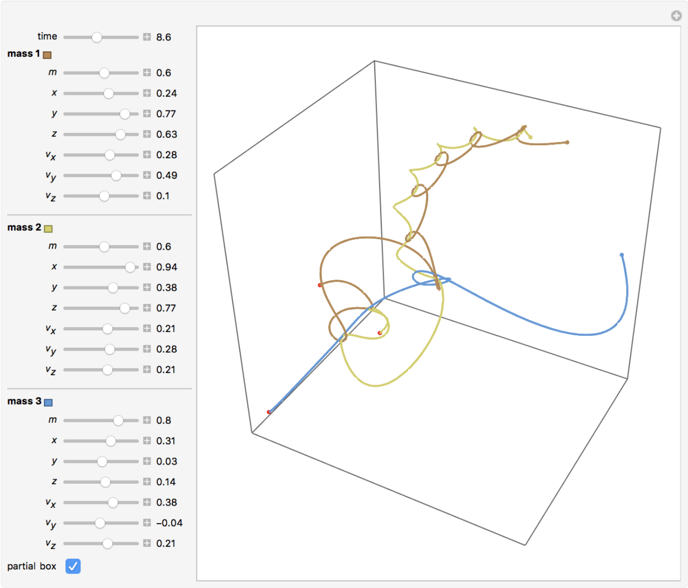


And here's a version in 3D where you can set all the positions and velocities in 3D:



If we just had two objects (a "two-body problem"), all that would ever happen is that they'd orbit each other in a simple ellipse. But adding a third object ("three-body problem") immediately allows dra-

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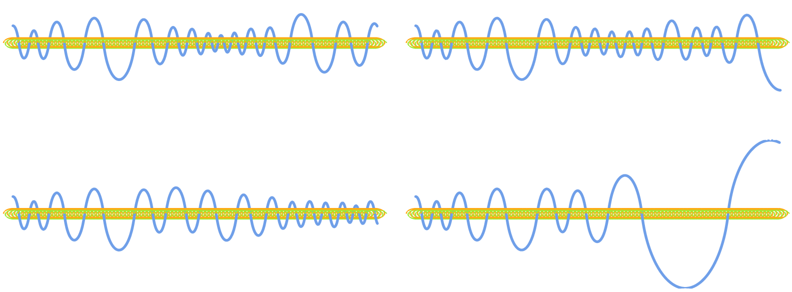
go their separate ways. Sometimes two form a binary system and the third goes separately. And sometimes all three make anything from an orderly arrangement to a complicated tangled mess.

The three-body problem turns out to be a classic example of the chaos-theory idea of sensitive dependence on initial conditions: in many situations, even the tiniest change in, say, the initial position of an object will be progressively amplified. And the result is that if one specifies the initial conditions by numbers (say, for coordinate positions), then the evolution of the system will effectively "excavate" more and more digits in these numbers.

Here's a particularly simple example. Imagine having a pair of objects in a simple elliptical orbit. Then a third object (assumed to have infinitesimally small mass) is started a certain distance above the plane of the ellipse. Gravity will make the third object oscillate back and forth through this plane forever. But the tricky thing is that the details of these oscillations depend arbitrarily sensitively on the details of the initial conditions.

This picture shows what happens when one starts that third object at one of four different coordinate positions that differ by one part in a billion. For a while, all of them follow what looks like exactly the same trajectory. But then they start to diverge, and eventually each of them does something completely different:



Plotting this in 3D (with the initial position $z(0)$ shown going into the page) we can see just how random things can get—even though each specific trajectory is precisely determined by the sequence of digits in the real number that represents its initial condition. (It's not trivial, by the way, to compute these pictures correctly; it requires using the arbitrary-precision number arithmetic of the Wolfram Language—and as time goes on more and more digits are needed.)

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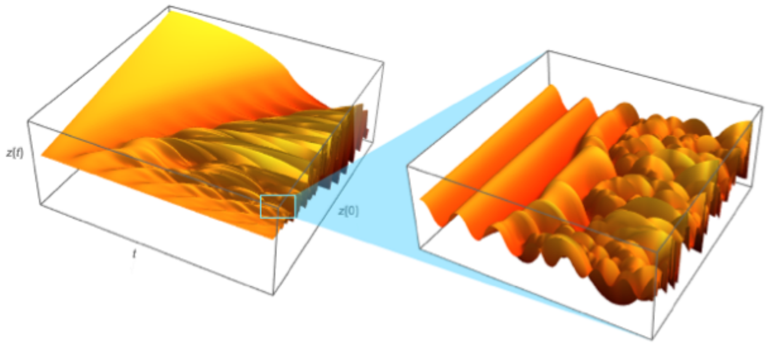

Not surprisingly, there's no simple formula that represents these results. But a few interesting things have been proved—for example that if one measures each oscillation by how many orbits are completed while it is happening, then one can get any sequence of integers one wants by choosing the initial conditions appropriately.

The two-body problem was solved in terms of mathematical formulas by Isaac Newton in 1687—as a highlight of his introduction of calculus. And in the 1700s and 1800s it was assumed that eventually someone would find the same kind of solution for the three-body problem. But by the end of the 1800s there were results (notably by Henri Poincaré) that suggested there couldn't be a solution in terms of at least certain kinds of functions.

It's still not proved that there can't be solutions in terms of any kind of known functions (much as even though there aren't algebraic solutions to quintic equations, there are ones in terms of elliptic or hypergeometric functions). But I strongly suspect that there can never, even in principle, be a complete solution to the three-body problem as an explicit formula.

Gravitational Computation

One can think of the time evolution of a system of masses interacting according to gravity as being a computation: you put in the initial conditions, and then you get out where the masses are after a certain time. But how sophisticated is this computation? For the two-body problem, it's fairly simple. In fact, however long the actual two-body system runs, one can always find the outcome just by plugging numbers into a straightforward formula.

But what about the three-body problem? The pictures above suggest a very different story. And indeed my guess is that the evolution of a three-body system can correspond to an arbitrarily sophisticated computation—and that with suitable initial conditions it should in

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fact be able, for example, to emulate any Turing machine, and thus act as a universal computer.

I've suspected computational universality in the three-body problem for about 35 years now. But it's a technically complicated thing to prove. Usually in studying computation we look at fundamentally discrete systems, like Turing machines or cellular automata. But the three-body problem is fundamentally continuous—and can for example make use of arbitrarily many digits in the real numbers it's given as initial conditions.

Still, at least from a formal point of view, one can set up initial conditions that have, say, a finite sequence of nonzero digits. Then one can look at the output from the evolution of the system, binning the results to get a sequence of discrete data (e.g. using ideas of symbolic dynamics). And then the question is whether by changing the initial conditions we can have the output sequence correspond to the result from any program we want—say one that shows which successive numbers are prime, or computes the digits of pi.

So what would it mean if we could prove this kind of computational universality? One thing it would mean is that three-body problem must be computationally irreducible, so there couldn't ever be a way to "shortcut"—say with a formula—the actual computation it does in getting a result. And another thing it means is that certain infinite-time questions—like whether a particular body can ever escape for any of a particular range of initial conditions—could in general be undecidable.

(There's a whole discussion about whether the three-body problem, because it works with real numbers, can compute more than a standard universal computer like a Turing machine, which only works with integers. Suffice it here to say that my strong suspicion is that it can't, at least if one insists that the initial conditions and the results can be expressed in finite symbolic terms.)

Random Orbits

How stable are the seemingly random trajectories in the three-body problem? Some are very sensitive to the details of the initial conditions, but others are quite robust. And for example, if one were designing a trajectory for a spacecraft, it seems perfectly possible that one could find a complex and seemingly random trajectory that would achieve some purpose one wants.

Are there cases where actual star or planetary systems will exhibit apparent randomness? There were undoubtedly examples even in the history of our own solar system. But because randomness tends to

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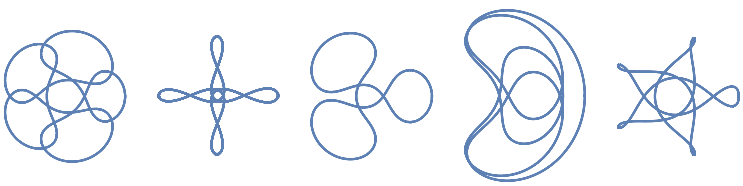
bring bodies into regions where they haven't been before, there's a higher chance of disruption by external effects—such as collisions—and so the apparent randomness probably doesn't typically last under "natural selection for solar systems" when there are many bodies in the system.

In the ever-difficult problem of working out whether something is of "intelligent origin", the three-body problem adds another twist—because it allows astronomical processes to show complexity just as a consequence of their intrinsic dynamics. If it is indeed possible to do arbitrary computation with a three-body system, then such a system could in principle be programmed to, say, generate the digits of pi, and perhaps make them visible in the light curve of a star. But often the system will show just as complex behavior from many different initial conditions—and one won't be able to tell whether the behavior has any element of "purpose".

Gravitational Engineering

Can one pick initial conditions for the three-body problem to achieve particular kinds of behavior? The answer is certainly yes. One example (already found by Lagrange in 1772) is to have the bodies on the corners of an equilateral triangle—which produces stable periodic behavior.

One can find other periodic configurations too:



And indeed, particularly if one allows more bodies, given some specified periodic trajectory, one can probably find (by fairly traditional gradient descent methods) initial conditions that will reproduce it, at least to some accuracy. (A notable example found in 1993 is just three bodies following a figure-eight orbit.)

But what about more-complex trajectories? Clearly, each set of initial conditions gives some kind of behavior. The question is whether it's useful.

The situation is similar to what I've encountered for a long time in studying simple programs like cellular automata: out there in the computational universe of possible programs, there's all kinds of rich and complex behavior. Now the issue is to "mine" those examples that are actually useful for something.

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In practice, I've done lots of "algorithm discovery" in the computational universe, setting up criteria and then searching huge numbers of possible programs to find ones that are useful. And I expect exactly the same can be done for gravitational systems like the three-body problem. It's really a question of formulating some purpose one's trying to achieve with the system; then one can just start searching, often quite exhaustively, for a case that achieves that purpose.

So how do black holes work in things like the three-body problem? The basic story is simple: so long as the bodies stay far enough apart, it doesn't matter whether they're black holes or just generic masses. But if they get close, there'll start to be relativistic effects, and that's where black holes will be important. Presumably, however, one can just set up a constraint that there should be no close approaches, and one will still be able to do plenty of gravitational engineering—with black holes or any other massive objects.

Where Can We Get Black Holes, Anyway?

If we're going to be able to do serious black hole engineering, we'd better have a serious source of black holes. It's not clear that our universe is going to cooperate on this. There are probably big black holes at the centers of galaxies (and that may be the rather unsatisfying answer to "what's the 'equilibrium' state" of a large number of self-gravitating objects). There's probably a decent population of black holes from collapsed massive stars—perhaps one per thousand stars or so, which means 100 million spread across our galaxy.

There's an important other point to mention about black holes: if current theories correctly graft certain aspects of quantum mechanics onto the classical physics of the Einstein equations, then any black hole will emit Hawking radiation, and will eventually evaporate away as a result. Star-sized black holes would have huge lifespans, but for less-massive black holes, the lifespan goes down, and for a black hole the mass of Halley's comet, the lifespan would be about a billion years.

What about tiny black holes? Hawking radiation suggests they should evaporate almost instantly: an electron-mass one should be gone in well under 10^{-100} seconds. (When I was 15 or so, I remember asking a distinguished physicist whether electrons could actually be black holes. He said it was a stupid idea, which probably it was. But in writing this blog I discovered that Einstein also considered this idea—though about 50 years before I did. And as it happens, in my network-based models, electrons do end up being made of "pure space", not so unlike black holes.)

Even if it's hard to get genuine gravitational black holes, one might wonder if there could at least be analogs that are easier to get. And in recent years there's been some success with making "sonic black holes"—that are at least a rough analog of gravitational black holes, but where it's sound, rather than light, that's trapped.

Finally, Black Hole Technology

OK, so we're now finally ready to talk about creating technology with black holes. I should say at the outset that I'm not at all happy with what I've managed to figure out. Lots of things I thought might work turn out simply to be impossible when one looks at them in the light of actual black hole physics. And some others, while perhaps interesting, require assembling large numbers of black holes, which seems almost absurdly infeasible in our universe—given how sparse at least larger black holes seem to be, with only perhaps 10^19 spread across our whole universe.

Time Travel to the Future

But let's say we just have one black hole. What can we do with it? One answer is to "bask in its time dilation"—or in some sense to use it to do "time travel to the future".

Special Relativity already exhibits the phenomenon of time dilation, in which time runs more slowly for an object that's moving quickly. General Relativity also messes around with the rate at which time runs. In particular, in a place with stronger gravity, time runs slower than in a place with weaker gravity. And so this means, for example, that as one goes further from the Earth, time runs slightly faster. (The clocks on GPS satellites are back-corrected for this—making them at least naively appear to "violate General Relativity".)

Near a black hole, strong gravity can make time run significantly more slowly. There's a nice example in the movie *Interstellar*, in which there's a planet orbiting at exactly the right distance from a black hole with exactly the right parameters—so that time runs much more slowly on the planet, but other gravitational effects there aren't too extreme.

In a sense, as soon as one has a way to make time locally run slower, one can do "time travel to the future". For the "traveler" a month might have elapsed—but outside it could have been a century. (It's worth mentioning that one can achieve the same kind of effect

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without gravity just by doing a trip in which one accelerates to close to the speed of light.)

Of course, even though this would allow "time travel to the future", it would give no way to get back. For that, one would need so-called closed timelike curves, which do in principle exist in solutions to the Einstein equations (notably, the one found by Kurt Gödel), but which don't seem to appear in any physically realizable case. (In a system determined by equations, a closed timelike curve is really less about "traveling in time" than it is about defining a consistency condition between what happens in the past and the future.)

Black-Hole-Mediated Travel

In science fiction, black holes and related phenomena tend to be a staple of faster-than-light travel. At a more mundane level, the kind of "gravity assist" maneuvers that real spacecraft do by swinging, say, around Jupiter could be done on a much larger scale if one could swing around a black hole—where the maximum achievable velocity would be essentially the speed of light.

In General Relativity, the only way to effectively go faster than light is to modify the structure of spacetime. For example, one can imagine a "wormhole" or tube that directly connects different places in space. In General Relativity there's no way to form such a wormhole if it doesn't already exist—but there's nothing to say such wormholes couldn't already have existed at the beginning of the universe. There is a problem, though, in maintaining an "open wormhole": the curvature of spacetime at the end would tend to create gravity that would make it collapse.

I don't know if it can be proved that there's no configuration of, say, orbiting black holes that would keep the wormhole open. One known way to keep it open is to introduce matter with special properties like negative energy density—which sounds implausible until you consider vacuum fluctuations in quantum field theory, inflationary-universe scenarios or dark-energy ideas.

Introducing exotic matter makes all sorts of new solutions possible for the Einstein equations. A notable example is the Alcubierre solution, which in some sense provides a different way to traverse space at any speed, effectively by warping the space.

Could there be a solution to the Einstein equations that allows something similar, without exotic matter? It hasn't been proved that it's impossible. And I suppose one could imagine some configuration of judiciously placed black holes that would make it possible.

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It's perhaps worth mentioning that in the models I've studied where the underlying structure of spacetime is a network with no predefined number of space dimensions, wormhole-like phenomena seem more natural—though insofar as the models reproduce General Relativity on large scales, this means such phenomena can't originate on those scales.

Energy Sources

It's easy to generate high energies with a black hole. Matter that spirals in towards the black hole will gain energy—and indeed, around stellar and larger black holes there's potentially an accretion disk that contains high-energy matter.

With rotating black holes, there are some additional energy phenomena. In the ergosphere, objects can gain energy at the expense of the black hole itself. This is relevant both in accelerating ordinary matter, and in producing "superradiance" where energy is added to waves, say of light, that pass through the ergosphere.

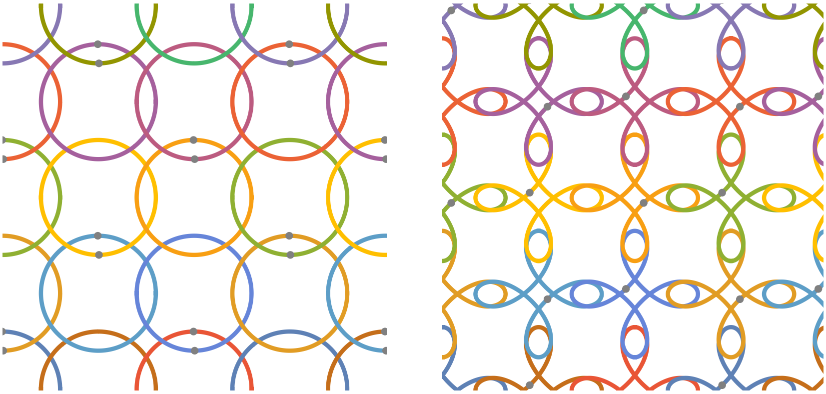
Can one do better with multiple black holes than a single one? I don't know. Maybe there's a configuration of orbiting black holes that's somehow optimized for imparting energy to matter—like a kind of particle accelerator made from black holes.

Gravitational Crystals

We saw earlier some of the complex trajectories that three bodies interacting through gravity can follow. But what kind of trajectories can we potentially "engineer", particularly with more bodies?

It's not too difficult to start with approximate trajectories and then do gradient descent (e.g. in Fourier space) to try to find trajectories that actually correspond, for example, to closed orbits. So can one for example find a "gravitational crystal" that consists of an infinite regular array of interacting gravitational bodies?

There are some mathematical tricks to apply—and one ends up having to use randomized search more than systematic gradient descent—but there do seem to be gravitational crystals to be found. Here are two potential examples that show a kind of checkerboard symmetry:



I suppose a "gravitational wall" like this might be good for stopping things that approach it. With the right parameters, it might be able to capture anything (perhaps up to some speed) that tries to cross it.

Given a "gravitational crystal", one can ask about implementing things like cellular automata on it. I don't know how to store "bits" for cellular automaton cells in lattices like these without disrupting the lattice too much, but I suspect there's a way. (Yes, classical gravity is reversible, so one would have to have reversible cellular automata, but there are plenty of those.)

What's shown here is something that's intended to be a regular, periodic "crystal". One can also potentially imagine creating a "random crystal" in which there's overall regularity, but at a small scale there's seemingly random motion. If one could make such a random crystal work, then it might provide a more robust "wall", less affected by outside perturbations.

Gravitational Shielding

Modularization is an important general technique in engineering because it lets one break a problem into parts and then solve each one separately. But for gravitational systems, it's hard to do modularization—because gravity is a large-range force, dropping off only gradually with distance.

And even with spinning black holes and the like, I don't know of any way to achieve the analog of gravitational shielding—though this changes if one introduces exotic matter that effectively has negative mass, or if, for example, every black hole has electric charge.

And without modularization, it's surely more difficult to create something technologically useful—because in effect one has to figure out everything at once. But it's certainly conceivable that by searching

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a space of possibilities one could find something—though without modularization it might look very complicated (as long-range simple programs, like combinators, tend to do), and it could be difficult even to tell what the system achieves without looking for specific properties one already knows.

What I'm Missing...

Having said all this, I suspect that there are big things I am missing—and that with the right ways of thinking, there'll end up being some spectacular kinds of technology that black holes make possible. And for all we know, once we figure this out we'll realize that an example of it has already existed in our universe for a billion years, whether of "natural" origin or not.

But for now, the discovery of gravitational radiation from merging black holes is a remarkable example of how something like the small equation Einstein wrote down for the gravitational field a hundred years ago can lead to such elaborate consequences. It's an impressive endorsement of the strength of theoretical science—and perhaps an inspiration to see just how small the rules might be to generate everything we see in our universe.

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