Persistent Structures in Elementary Cellular Automaton Rule 146

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Two types of spacetime structures found in the evolution of rule 146 are presented: very large white triangles, called monoliths, and persistent structures formed by sequential white triangles of even width. The monoliths are discussed in terms of large fluctuations away from equilibrium in a many-body system. The persistent structures are derived by highlighting the nonadditive portions of the rule 146 evolution, and some of their statistical properties are presented. The use of these structures for directing information flow in this class 3 rule are discussed.

1. Introduction

The Principle of Computational Equivalence (PCE) in *A New Kind of Science* (NKS) [1] makes the claim that class 3 elementary cellular automaton (ECA) systems are universal computers. To date, no examples of universal class 3 systems are known. Therefore, understanding the computational abilities of class 3 ECAs is a key subject of current interest in the field of NKS. This paper discusses some interesting spacetime structures that arise in the evolution of ECA rule 146, and how it compares to the closely related and well understood rule 90.

2. Class 3 Rules

Rule 30 is an example of a "class 3" ECA rule, meaning it is complex, but with little discernible structure. Class 3 rules show largely random behavior, and since they are irreducible we do not know how to replicate what they produce other than by simply running them. Figure 1 shows how several class 3 rules behave from a random initial condition. The PCE [1] claims that class 3 rules are in fact universal, meaning any computation can be carried out using, for example, rule 30. However, to date, no examples are known. So, studying class 3 rules is a big focus in NKS. Constructing a proof that a particular class 3 rule is universal would be evidence in support of the PCE.

This paper focuses on ECA rule 146, shown in the bottom right-hand corner of Figure 1. It has been known since the early 1980s that rule 146 is actually in a family of several class 3 rules that generate similar particle-like structures, including rules 18 (top left), 126 (bottom middle), and 122 (not shown) [2]. More recent work in theoretical computer science has shown that the limit languages of rules 18, 126, and 146 are closely related [3]. The language complexity of rule 146 has also recently been studied in detail [4]. These class 3 rules are more tractable to study than rule 30, for example, because they are closely related to rule 90 [4] (see Section 4). Since rule 90 is additive, it is easier to analyze, and we can view rules like 146 as a slight deviation from perfectly additive behavior.

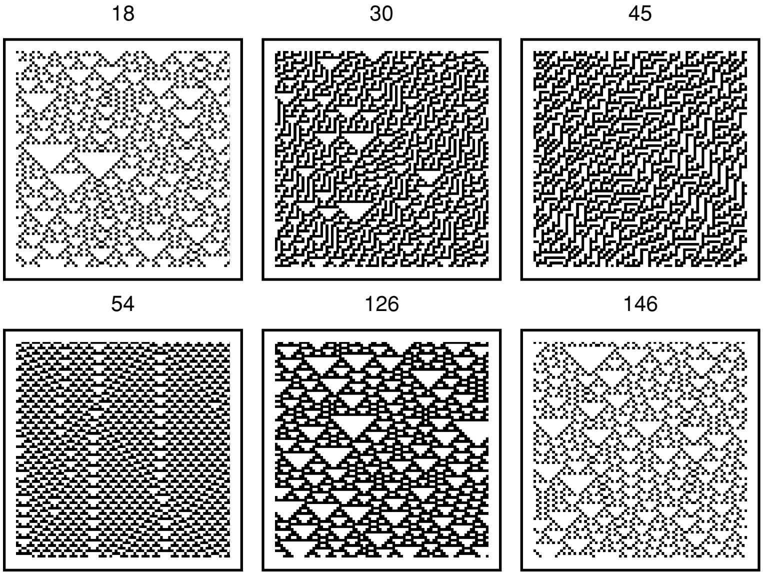


Figure 1. Class 3 ECA rules. Each rule is run from a randomly chosen initial condition consisting of 200 black and white cells, with equal probability of being either black or white.

3. Monoliths

I first saw the surprising behavior in the evolution of ECA rule 146 depicted in Figure 2 while at the NKS Summer School 2004 [5]. Note the unusually large triangles coming out in a regular array, decreasing in size with time. I dubbed these unusually large triangles "monoliths", and have continued to study them ever since.

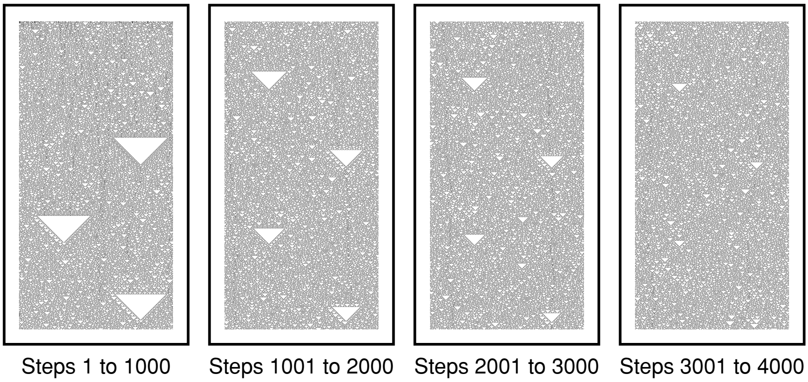


Figure 2. ECA rule 146 from a random initial condition of width 500. The large white triangles are called monoliths.

Monoliths are interesting to me because I think of them as fluctuations. Figure 3 shows the density (number of black cells divided by the total number of cells) during the evolution of rule 146. On the left, monoliths appear around step 400, and on the right no monoliths appear during the evolution. The large white triangles of the monoliths cause a large, sudden dip in the density. The density then recovers as the monolith dies away, followed by another large dip when the second monolith appears. The process continues with the monoliths and accompanying density drops getting smaller and smaller.

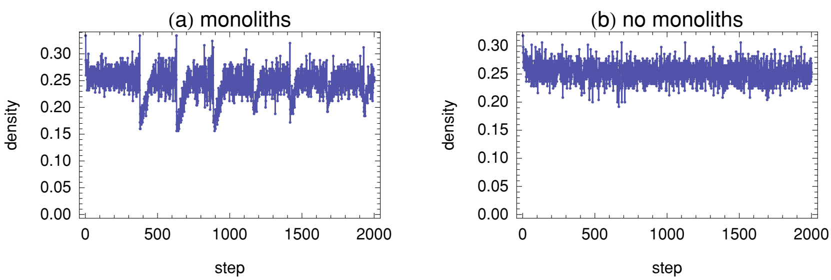


Figure 3. Density of black cells during the evolution of rule 146 where (a) monoliths appear and (b) monoliths do not appear. The presence of monoliths is marked by larger density fluctuations than are normally present in the system.

It is interesting to note that there is little indication from the density that monoliths will appear. That is, the "mean field" description of the system does not seem to contain sufficient information to predict the onset of these large fluctuations. From the density viewpoint, the system looks like it is in equilibrium, but suddenly undergoes large, spontaneous fluctuations. So, monoliths represent large fluctuations in a system that is apparently at equilibrium, which ties into the field of nonequilibrium physics and phase transitions [6]. Are there similar effects in real physical systems?

Understanding monoliths may help us understand how to use rule 146 (and possibly other related class 3 rules) to do computations. The monoliths represent a break from the usual "bubbling around" during the evolution of rule 146 when run from random initial conditions. Structures like monoliths are potentially useful in controlling information flow through the system.

4. Persistent Structures

One approach to understanding rule 146 is to solve a simpler problem. Notice in Figure 4 that the rule tables for rules 146 and 90 [7] differ in only three spots. In particular, the rules give different outputs only when there are adjacent black cells in the input. When there are no adjacent black cells, the rules give the same output.

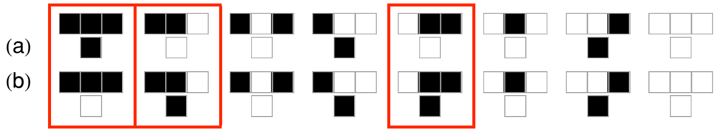


Figure 4. Comparison of rule tables for rules (a) 146 and (b) 90, with the differences boxed in red.

As shown in Figure 5, the evolution from a single black cell is identical for rules 90 and 146 because adjacent black cells are never generated in this case.

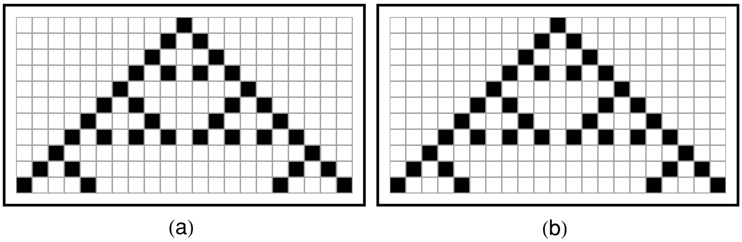


Figure 5. Evolution of rules (a) 90 and (b) 146 from a single black cell. The evolutions from this initial condition are identical, because adjacent black cells are never generated.

Under what conditions do adjacent black cells appear in rules 90 or 146? Figure 6 shows that black cells separated by even runs of white cells evolve to produce adjacent black cells. Once the adjacent black cells appear, the rules evolve differently, just as we saw must be the case from the rule tables. So we can conclude that even runs of white cells are responsible for adjacent black cells.

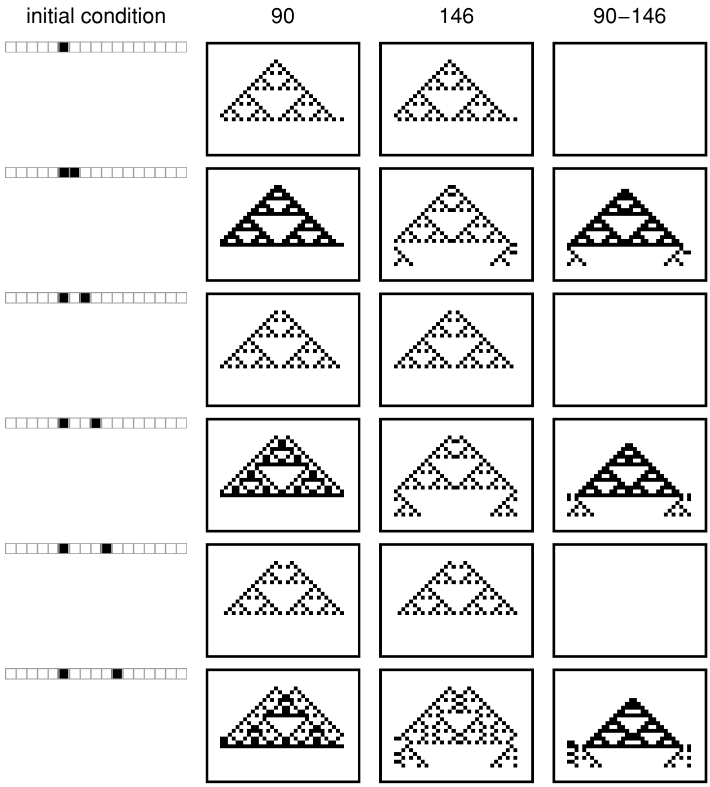


Figure 6. Comparison of rules 90 and 146 for initial conditions containing black cells separated by an increasing number of white cells.

If we highlight the even runs in the evolution of rule 146, we can see how it differs from 90. In Figure 7, even runs of white cells are highlighted in red, and adjacent black cells in blue. The result is very interesting! We can see a series of even-run triangles (red), connected by pairs of ones (blue).

The surprising fact here is that the red even-run triangles are connected in time. That is, a new even-run triangle is always formed when adjacent black cells appear at the base of the previous one. (We can think of the adjacent black cells at the base being an even run of white cells of length zero.) This is an emergent property of the system, since the formation of each even-run triangle depends on that triangle's local environment.

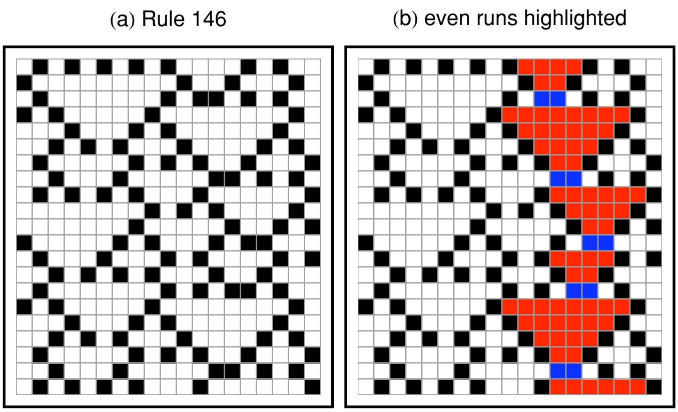


Figure 7. Evolution of (a) rule 146 and the same evolution (b) with even runs of white cells highlighted in red, and adjacent black cells highlighted in blue.

When viewed in the context of a longer evolution, as shown in Figure 8, it is clear that the chain of even-run triangles forms a *persistent structure*. That is, a structure that is localized in space, and persistent in time. Here, the initial condition has been carefully set up to contain only a single even run in the center, and odd runs everywhere else.

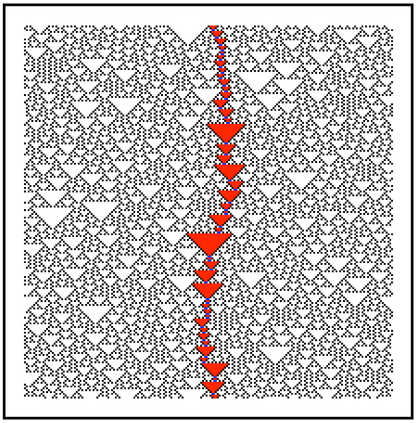


Figure 8. Rule 146 run from a random initial condition with only a single even run of white cells in the center, and odd runs everywhere else. The even runs of white cells are highlighted in red.

Figure 9 shows that when run from a randomly chosen initial condition with no restrictions on how many even runs may appear, rule 146 forms many of these persistent structures. Between the structures, we know there must only be odd runs of white cells and no adjacent black cells. Therefore, the evolution between the structures is according to rule 90. So, rule 146 evolves the same as rule 90 with interruptions, or perturbations, in the form of these localized structures.

Note that the structures appear to be annihilating with each other in pairs. Structures with similar pairwise annihilation and random walk behavior were found in ECA rule 18 by Grassberger in 1984 [2].

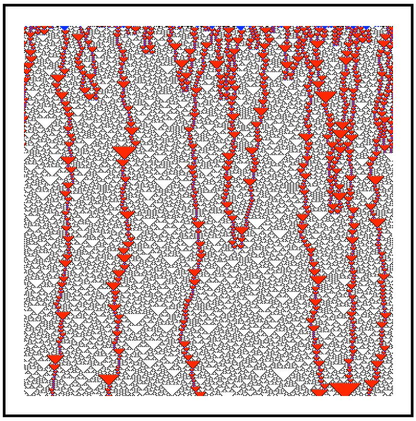


Figure 9. Rule 146 run from a randomly chosen initial condition with no restriction on how many even runs may appear. As in Figure 8, the even runs of white cells are highlighted in red, and even runs of black cells are highlighted in blue.

If we take the evolution showing the monoliths in Figure 2, and now highlight the even runs as in Figures 7 through 9, we get the result shown in Figure 10. The first couple of monoliths have multiple persistent structures feeding into them.

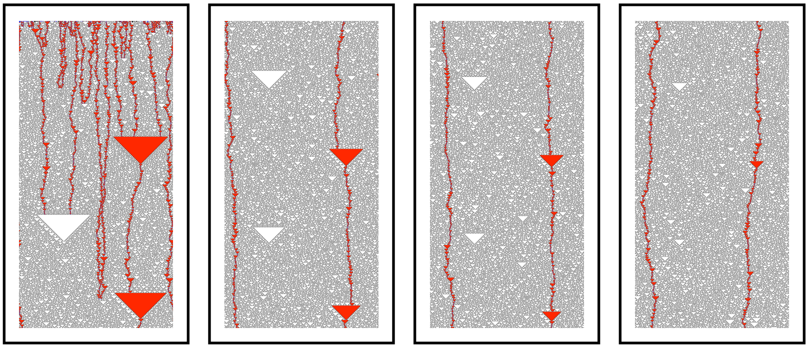


Figure 10. The evolution in Figure 2 with even runs of white cells highlighted in red and even runs of black cells highlighted in blue.

5. Properties of Persistent Structures

How long do the structures persist? Figure 11 shows that if there is only one structure, it seems to persist forever (as long as there is not a total annihilation).

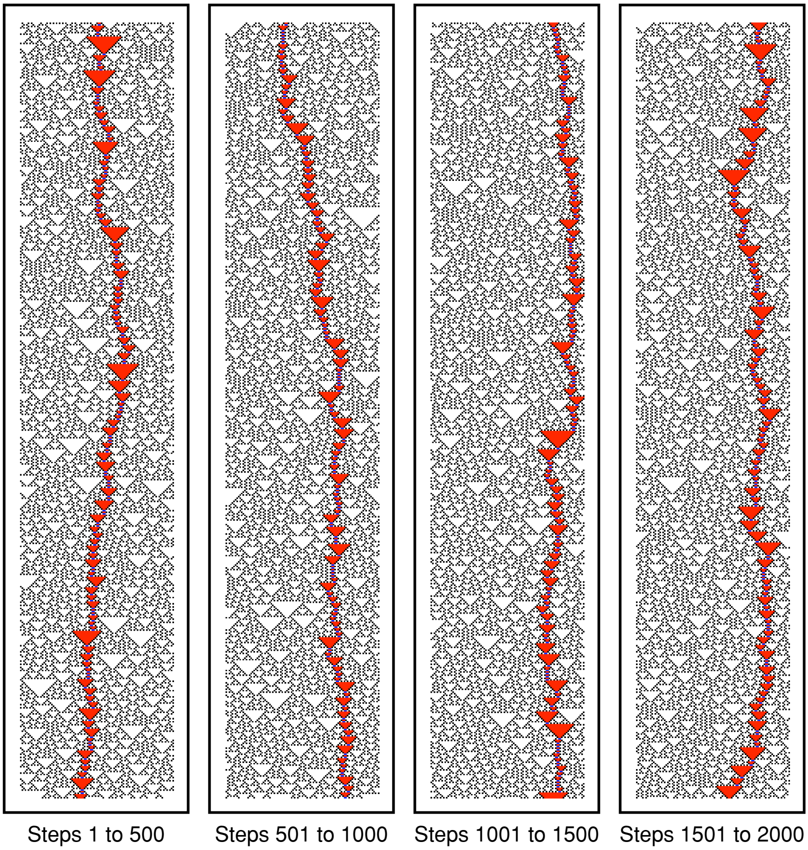
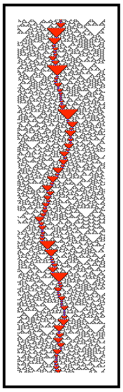


Figure 11. Continuous evolution of a single persistent structure in rule 146 (highlighted in red).

Figure 12 shows the distribution of widths of even runs on a single persistent structure. The blue line is an exponential fit to the distribution.

Figure 13 shows the position of the persistent structure over time. It would be interesting to study how this movement differs from a random walk.



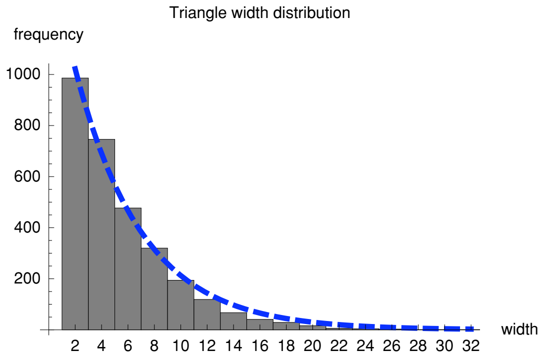


Figure 12. Distribution of run lengths for even runs in the evolution of rule 146 with a single persistent structure (left). The blue line shows an exponential fit to the distribution.

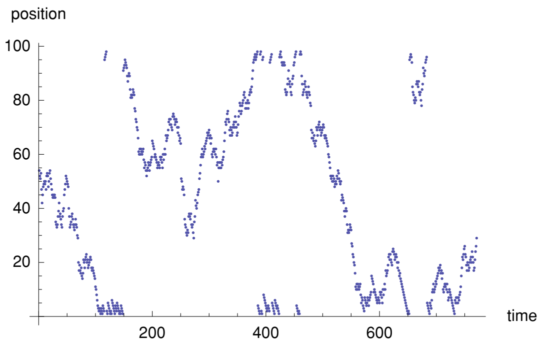
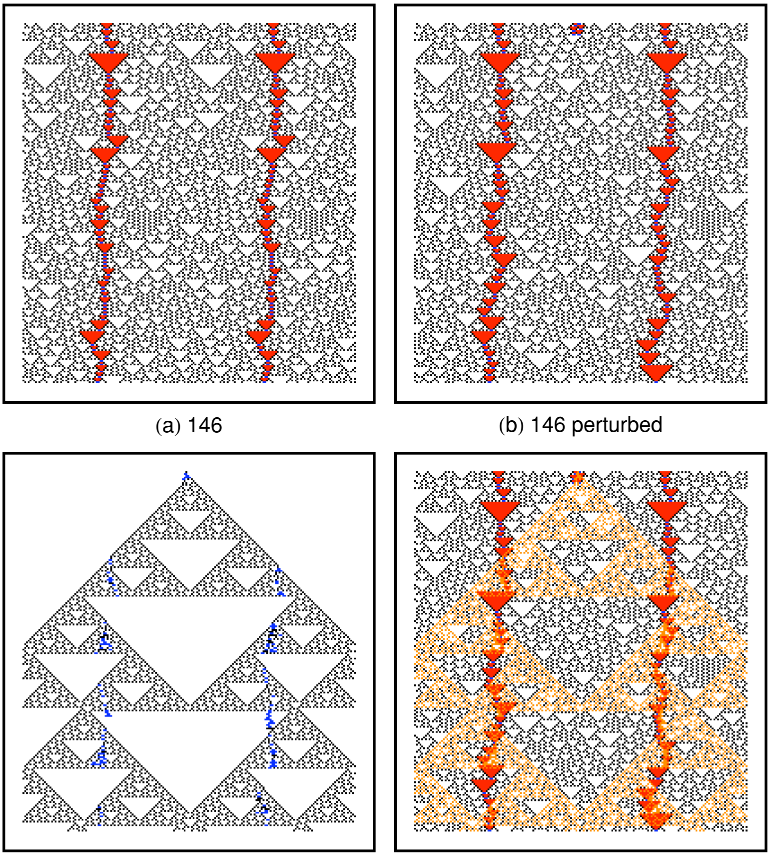


Figure 13. Position of a single persistent structure during the evolution of rule 146 with a width of 100.

How do the persistent structures affect the flow of information in the system? Figure 14 shows how the spread of perturbations during the evolution of rule 146 interacts with the structures. Starting with two structures (a), perturb a site between them (b). Then show the difference between the two evolutions (c). The difference pattern is nearly the same as for rule 90 in Figure 15(c), with the exception of two regions of runs of black cells. Overlaying the difference pattern on the perturbed evolution (d) shows that the non-90 regions follow the persistent structures.

It looks like there are two modes of information transmission in rule 146: the linear (rule 90) part and the nonlinear part. The linear part is what you get from an additive system (rule 90). It moves at light speed, and gets transmitted through the persistent structures.

The nonlinear part is carried by the structures at sublight speed. There is an obvious analogy with light and particles here. Particles travel at sublight speed because they have mass. Here, the equivalent of mass would be even runs of white cells.



(a) 146 (b) 146 perturbed

(c) 146 difference (a) and (b) (d) 146 difference overlayed

Figure 14. The interaction of persistent structures in the evolution of rule 146 with damage fronts. (a) Start with an evolution producing just two structures. (b) The evolution after perturbing a site between the boundaries of the two initial structures in (a). (c) The difference between the evolutions in (a) and (b), with even runs of black cells highlighted in blue. (d) The difference pattern in (c) overlayed in orange on the evolution in (b).

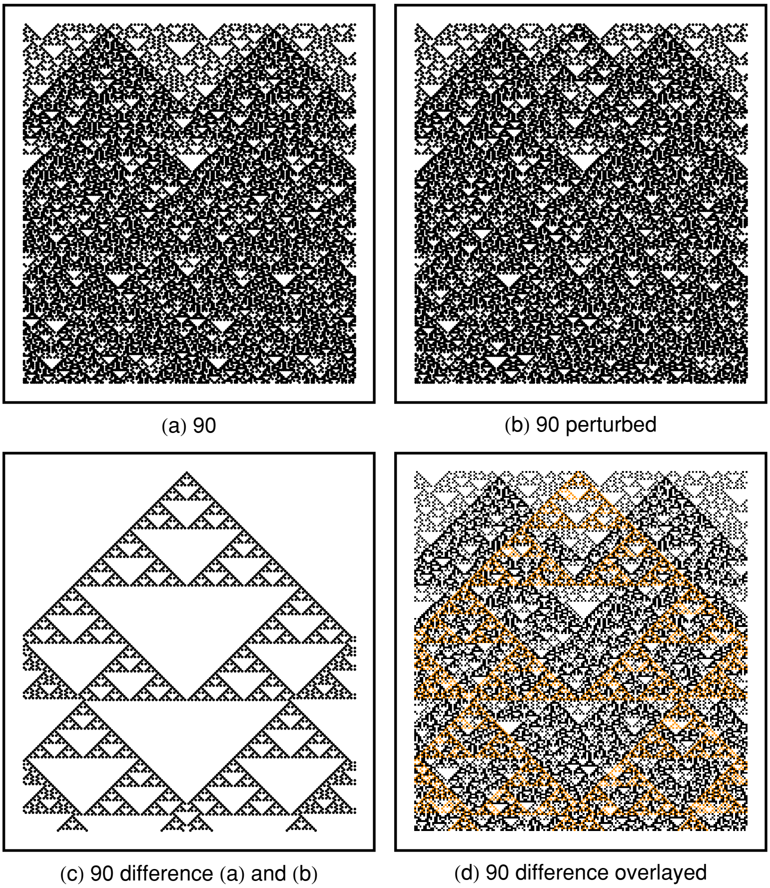


Figure 15. The evolution of rule 90 with (a) through (d) the same as in Figure 14.

6. Conclusions

In conclusion, rule 146 is a class 3 rule that generates unusually large triangles called monoliths. In order to understand rule 146 and how to compute with it, we compare it to rule 90. We find from comparing the rule tables of 146 and 90 that the rules differ only when there are adjacent black cells present. Adjacent black cells, in turn, appear at the base of triangles of even width. Highlighting even runs of white cells in the evolution of rule 146 reveals structures that are localized in space and persistent in time. The structures seem to do random walks, and annihilate in pairs. Perturbing the region between two structures shows that information is transmitted at light speed between and through the structures, but leaves a nonlinear component following the paths of the structures.

References

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