A New Game of Three-Dimensional Life

Carter Bays
University of South Carolina, Columbia, SC 29208 USA

The three-dimensional cellular automaton known as *Life* was first re-
ported in references [1] and [2]; at that time, two rules were described. These
rules (also called "games of Life") are known as Life 4555 and Life 5766, where
the first two numbers give the "safe environment" range (the number of live
neighbor cells, represented as unit cubes, that must touch a currently live
cell so that it will remain alive next generation) and the second two numbers
specify the "fertility" range (the number of live touching neighbors required
to bring a currently dead cell to life next generation). It is possible to give a
somewhat formal definition to rules that qualify as "games of Life"; this was
done in [1] and the definition is repeated here.

Definition 1. *A rule $E_l E_u F_l F_u$ (E and F are the environment and fertility
ranges; l and u give lower and upper limits) defines a "Game of Life" if and
only if both of the following are true.*

1. *A glider (i.e., a translating oscillating form) must exist and must oc-
   cur "naturally" if we apply $E_l E_u F_l F_u$ repeatedly to primordial soup
   configurations (random initial configurations of live cells).*

2. *All primordial soup experiments, when subjected to the rule $E_l E_u F_l F_u$
   must exhibit bounded growth.*

Of course, our definition is subject to interpretation—we do not state
how many experiments must be performed before concluding that no glider
exists and, like the chimpanzee who produces Hamlet, anything can happen
if given enough time. But the time we are talking about here is the time
available on a digital computer.

It is somewhat surprising that out of more than 100,000 possible rules,
only 4555 and 5766 were found to satisfy the above definition. However,
it was pointed out (see [1]) that rule 5655 appeared to be promising, but
that "an exhaustive search has revealed that this rule supports no naturally
occurring glider."

NUMBER ALIVE =   18   20

GENERATION =   1   2   +Z

LIFE 5655 GLIDER
TWO VIEWS

1   2   ↓ -Y

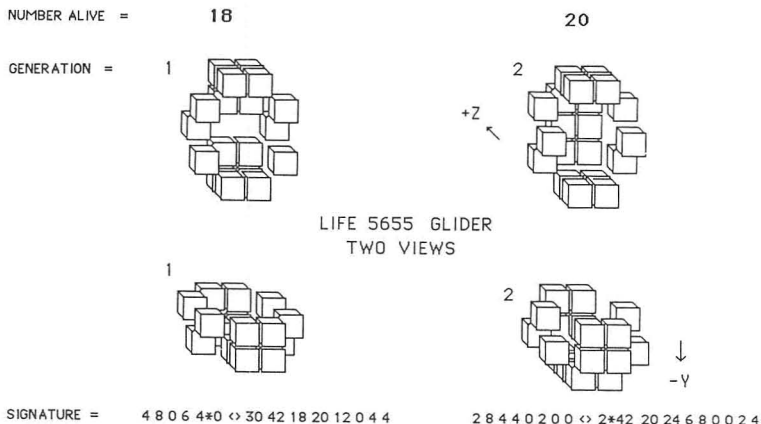SIGNATURE =   4 8 0 6 4∗0 ⟨⟩ 30 42 18 20 12 0 4 4   2 8 4 4 0 2 0 0 ⟨⟩ 2∗42 20 24 6 8 0 0 2 4

Figure 1: Two views of the 5655 glider are depicted. The object has a period of 2; the third generation repeats the first, but the glider has moved one unit in a direction parallel to a coordinate axis. The view at the top is moving in the $+z$ direction (away from viewer); the bottom version is moving in the $-y$ direction. The signature (see [3]) for each phase is given at the bottom. "$A * B$" means that there are A consecutive occurrences of B.

This statement was, at best, premature since only smaller configurations were tested (fewer than 11 live cells). Indeed, a glider has now been discovered for the rule 5655 (see figure 1) and, since the residue for this rule is even more sparse than for Life 4555 (i.e., unbounded growth is not a possibility), the rule 5655 (hereafter called "Life 5655") can be added to the (short) list of rules that define games of three-dimensional life.

A couple of caveats are in order. The Life 5655 glider is extremely rare. To find it, the program described in [4] was utilized. Initially, purely random configurations were tested. These configurations involved starting patterns that contained up to (approximately) 50 cells. Several million experiments were run without success. However, it was surmised that if a glider did exist for 5655, it more than likely exhibited some form of symmetry. The most logical symmetry to try was the same type appearing in the gliders for 4555 and 5766—namely, reflective in a plane parallel to one of the coordinate planes, such as $z = 0$. In fact, when this type of symmetry was imposed upon small initial primordial soup configurations, the gliders for 4555 and 5766 appeared about 30 to 100 times as often as when no symmetry was imposed. After about 30 million experiments, all involving the "symmetric soup" just described, the glider shown in figure 1 had appeared seven times. It is impossible to extrapolate accurately the number of non-symmetric soup experiments that would have been required to produce just one appearance of the glider, but 20 to 500 million is probably a reasonable guess.
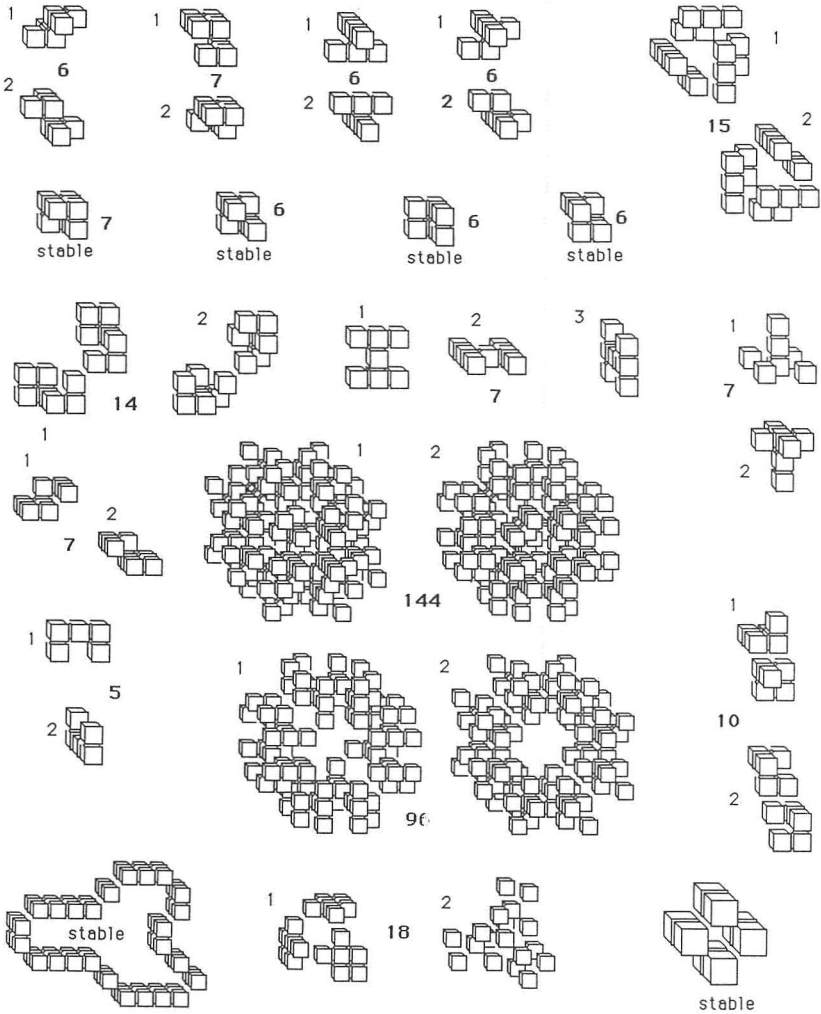
Figure 2: A representative sample of oscillators for Life 5655 is shown (see [1], figure 3 for additional examples). Most small oscillators for Life 5655 have a period of 2, although it is probable that a more thorough investigation will turn up some unusual longer period oscillators. Stable forms appear to be rather rare; forms like the stable object at the lower left must be carefully constructed. It is interesting to note that the (slightly larger) stable object at the lower right is stable for all three Life rules—5766, 4555, and 5655.

The fact that this glider is so rare means that it is virtually certain that no "glider gun" exists for 5655 (a "glider gun" is a carefully constructed configuration that spews out an endless supply of gliders). Nevertheless, Life 5655 does exhibit many interesting oscillators (see figure 2; see also [1], figure 3). Since Life 5655 has been shown to be "worthy of the name" [2] more work needs to be done with this rule—in particular a search for small oscillators with periods higher than 2. The oscillators found so far seem to exhibit characteristics quite different from those for Life 4555 and Life 5766. Perhaps their unusual nature will help compensate for the lack of a glider gun.

Work is currently being done on some of the rules that were mentioned in [1] but rejected. The "symmetric soup" experiments suggest that if unknown gliders do exist, they have a much better chance of being discovered when we can perform experiments at a rate that is in essence more than 1000 times faster than those that initially produced the gliders for Life 4555 and Life 5766.

References

[1] Carter Bays, "Candidates for the Game of Life in Three Dimensions," *Complex Systems*, **1** (1987) 373–400.

[2] A. K. Dewdney, "The Game Life Acquires Some Successors in Three Dimensions," *Scientific American*, **224**(2) (1987) 112–118.

[3] Carter Bays, "Patterns for Simple Cellular Automata in a Universe of Dense Packed Spheres," *Complex Systems*, **1** (1987) 853–875.

[4] Carter Bays, "The Discovery of a New Glider for the Game of Three-Dimensional Life," *Complex Systems*, **4** (1990) 599–602.