Complexity as a Linguistic Variable

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One of the problematic aspects of "complexity" is still its definition. We encounter many definitions and criteria for a system being complex from different points of view, some of which give rise to contradictory results: for a given system some definitions deduce that the system is complex, but others lead to the opposite result. An attempt is made to deal with the concept of complexity from a new viewpoint. A look at the fuzzy logic approach is suggested as an appropriate way to better understand what complexity is. The new idea is to find a definition of complexity based on linguistic variables (one of the foundations of fuzzy logic). To do this, we distinguish "system" from the corresponding "model" as a specific interpretation of the "system". We conclude that "complex" is a most appropriate adjective for "models" but not for "systems". Far from claiming to have come closer to the ultimate definition of complexity, hereby only a window is opened to see complex models in a new way.

1. Introduction

Consider the following question: can one specify a set with precise and sharp boundaries whose members are complex systems? To do this, it seems we should first realize what a complex system is. The definition of complex systems is one of the problematic aspects of a general theory of systems since one encounters many definitions and explanations of complexity derived many different points of view. Some give rise to contradictory results, some are cyclic, and some even try to show that there is no complexity theory at all.

For instance, based on some descriptions which portend that a complex system is composed of many parts that are interrelated in a complicated manner (e.g. [14] and references therein), most chaotic systems cannot be complex because they have only few variables (states). On the other hand, in terms of Prigogine's view that complexity is a property of systems that for given boundary conditions have more than one possible solution [16], the chaotic systems are complex. For example, the famous Belousov-Zhabotinsky reaction with few variables is considered as a complex system [16]. In addition, the chaotic logistic map, which is a one variable system (model), for not infinitely exactly specified initial conditions has more than one solution and therefore is complex in the sense of Prigogine. Hence in a particular system (model), we already have obtained two contradictory results.

Recently the following definition of a complex system has been proposed: a "complex system" is a system in which networks of components with no central control and simple rules of operation give rise to complex collective behavior, sophisticated information processing, and adaptation via learning or evolution [12]. This definition is cyclic and we are still not able to understand what exactly a complex is with properties that give rise to complex collective behavior: for we do not know yet what complex behavior is. So this latter concept needs a precise definition. Moreover, all the components of the definition should be clarified, that is, we need to know what exactly is central control, simple rules, information, and evolution. Thus, this definition needs complete modification.

It might seem that by finding an appropriate definition of simplicity we would be able to obtain an exact definition of complexity. But so only if we accept that "simplicity" and "complexity" are two opposite concepts (e.g. [7]). Moreover, even when assuming this, we again encounter many definitions and criteria for simplicity from different points of view. The concept of simplicity was for instance defined as "degree of falsification" by Popper [15], while many authors define it in other ways (e.g. see [4]). Therefore, it seems that this route does not lead us to finding a definition of complexity either. Furthermore, some definitions of simplicity can be applied only to whole theories to judge whether they are simple or not (e.g. Popper's falsifiability approach), but we also need to judge the simplicity of a given model (e.g. in a turbulence model). It must also be noted that many definitions of simplicity in the philosophical literature have never been about an abstract "lack of complexity" but rather attempted to redefine simplicity in terms of pragmatic virtues [5].

Another interesting and important approach to defining complexity is "complexity as difficulty of modeling" [6, 3]. This school believes that complexity can be defined as "the property of a language expression which makes it difficult to formulate its overall behavior even when given almost complete information about its atomic components and their inter-relations" [6]. According to this view a simple model is one that is easy to build, test, understand, and analyze [2]. Using this approach one must conclude that the notion of a system being complex is misleading in its own right since the "difficulty of modeling" does not primarily depend on some inherent properties of real world entities but also on the observer [1]. Meanwhile, the traditional tools of complexity measurement (e.g. entropy) then vanish from view because calling a system "essentially complex" is meaningless and leads one to conclude that "there is no complexity theory" [8]. While it seems that based on this definition, it still remains possible to compare the complexity of some given systems (models) with each other, it is not clear what the exact criterion of comparison is in view of the fact that the difficulty of modeling depends on the modeler and her or his abilities. In addition, the difficulty of modeling (i.e. complexity) is subject to change even for one given behavior. Although the first part of the last definition is important and we shall expand it in this note, the part which considers "complexity as difficulty of modeling" is unacceptable since it could not be defined clearly.

In the following we shall try to employ fuzzy logic to deal with the concept of complexity. Here the nature of natural language plays an important role because fuzzy logic can be seen as an attempt to focus on two outstanding human's capabilities: (i) that of conversation, reasoning and making rational decisions in an environment of imprecision, uncertainty, incompleteness and imperfect of information; and (ii) that of performing a wide variety of physical and mental tasks without involving any numerical computations [21]. In other words, we shall rely on the fact that it is significant to say that fuzzy logic is "computing with words" rather than "computing with numbers" [22, 28].

This work is organized as follows: in the next section some preliminaries necessary to deal with the fuzzy logic approach are presented. In section 3, a new viewpoint to define complexity based on the fuzzy logic is proposed and in section 4, a more general point to distinguish "complex models" from "complex systems" is offered. Finally in the last section, some remarks on the immediate future of complexity are tried.

2. A Brief Review of the Fuzzy Logic

The notion of fuzzy logic was born with the definition of fuzzy sets in a paper by Zadeh in 1965 [20]. He concentrated on those classes of objects in the real physical world having not precisely defined criteria of membership. For instance, when we refer to "the class (set) of young men", or "the class of real numbers much greater than five", then these sets do not have sharp and precise boundaries. Whereas, when we refer to "the class of men whose age lies between 18 and 30", or "the class of real numbers which are greater than five", these sets have sharp boundaries and we are able to specify their members even when this set is infinite. This distinction can be generalized to "events" [27, 25]. For example, consider two propositions "tomorrow is a cold day" and "the probability that the real number x lies between 3 and 4 is very likely". Obviously, the phrase "a cold day" in the first proposition does not have a precise definition. Also, the probability "very likely" in the second proposition does not fulfill sharply defined criteria, however, the phrase "x lies between 4 and 5" has a sharp boundary. Therefore, the classes of "young men" and "real numbers much greater than five" are both fuzzy rather than ordinary (crisp). In addition, the event "a cold day" is a fuzzy event and the probability "very likely" is a fuzzy probability [27]. More formally [20]:

Definition 1. A "*fuzzy set*" A in U (a universe of discourse) is characterized by a membership function $\mu_A(u)$, where u denotes a generic value of U, which associates with each point in U a real number in the interval [0, 1]. Here each value of $\mu_A(u)$ at u represents the "grade of membership" of u in A.

It is noted that if A is a non-fuzzy (ordinary or crisp) set in the universe of discourse U, then the membership function $\mu_A(u)$ is equal to 1 when u belongs to A, and is equal to 0 when u does not belong to A. In other words, in case of an ordinary set, the membership function $\mu_A(u)$ is a map from U into the set $\{0, 1\}$, but in the case of a fuzzy set, the membership function $\mu_A(u)$ is a map from U into the unique closed interval [0, 1].

Strictly speaking, fuzzy logic is a precise logic of imprecision and approximate reasoning based on two pivotal notions: graduation and granulation [21]. More concretely, in this logic everything is allowed to be graduated, that is, to be a matter of degree; and everything is allowed to be granulated, that is, a fuzzy set of points has the form of a clump of elements held together by similarity. More importantly, the concept of "graduated granulation" (i.e. fuzzy granulation) is a unique characteristic of fuzzy logic. It is inspired by the unique way in which we humans deal with complexity and imprecision [21].

An instance of granulation is the concept of a "linguistic variable" [21]. Informally speaking, a linguistic variable is a variable whose values are words or sentences rather than numerical entities [23]. For example, "height" can be considered as a linguistic variable if its values are "tall, quite tall, very tall, not tall, short, very short, not very tall and not very short, etc," rather than "190 cm, 186 cm, 198 cm, 170 cm, 162 cm, 140 cm, 167 cm, etc." Words have fuzzy denotations and much of human knowledge is described in a natural language [22, 28, 29]. In addition, much in human decision-making is based on a natural language rather than computing with numbers. For example, to pass through a given passage, a mobile robot must be programmed, and to this end needs all the exact data (initial conditions) such as the street width, velocity or acceleration of other cars, friction coefficients, wind velocity and wind direction, etc.; and all these data should remain constant during its operation, or else all changes in data should be specified in advance. If another vehicle suddenly increases its own speed heading towards it, then it will not be able to pass through the passage successfully due to the abrupt change in initial conditions. In other words, the robot is not able to create new information and divine a solution to adapt itself. By contrast, the mentioned act is one the simplest daily acts we do without any numerical data and information.

A natural language is a system for describing perceptions; however, perceptions are intrinsically imprecise, and therefore natural languages are imprecise since everything is or is allowed to be a matter of degree [21]. To witness, we humans characterize in everyday discourse the degree of truth of a statement by expressions such as "true, very true, more or less true, false, essentially false, etc," rather than by bivalent truth values in which every proposition can only be true or false. Indeed in fuzzy logic "truth" is considered as a linguistic variable [24], that is, as a variable whose values are words which cover an interval between false (say zero) and true (say one).

3. Complexity = a Linguistic Variable

In the preceding section we presented a brief review of the origin of fuzzy logic by explaining informally the concept of a "linguistic variable". In this section we shall define "complexity as a linguistic variable" and hence first need to clarify formally here what we mean by a linguistic variable. To this end we first must say what we mean by a "variable" [23] and by a "fuzzy variable" [24], in order then to define complexity.

Definition 2. A "*variable*" is characterized by a triple $(X, U, R(X; u))$ in which X denotes the name of the variable, U is a universe of discourse which can be finite or infinite, u denotes a generic name for the elements of U, and $R(X; u)$ is a subset of U representing a restriction on the values of u imposed by X.

Remark: A "variable" is associated with an "assignment equation" $x = u : R(X)$, or equivalently, $x = u$, $u \in R(X)$, which represents the assignment of a value u to x subject to the restriction $R(X)$. It should be noted that since this restriction is an ordinary subset of U, the assignment equation is satisfied if and only if $u \in R(X)$.

To illustrate: "height" can be considered as a variable (i.e. $X = \textit{height}$). In this case, the universe of discourse can be considered as the set of integer numbers with the unit of centimeters (cm). Finally, the restriction on the values of the universe of discourse imposed by "height" can be a subset of integer numbers which are less than 300 cm.

It is convenient to generalize the concept of a "variable" to a "fuzzy variable" [24]:

Definition 3. A "*fuzzy variable*" is characterized by a triple $(X, U, R(X; u))$, in which X denotes the name of the variable, U is a universe of discourse which can be finite or infinite, u denotes a generic name for the elements of U, and $R(X; u)$ is a fuzzy subset of U representing a fuzzy restriction on the values of u imposed by X.

Remark: Unlike the concept of "variable" in which the assignment equation is satisfied if and only if $u \in R(X)$, in the case of a "fuzzy variable" it is meaningful to define the degree to which the equation is satisfied. In this regard, the degree to which the assignment equation is satisfied will be determined by the compatibility of u with $R(X)$, denoted by $c(u)$ and defined by $c(u) = \mu_{R(X)}(u)$, where $\mu_{R(X)}(u)$ denotes the grade of membership of $u \in U$ in the restriction $R(X)$.

To illustrate: "height" can be seen as a fuzzy variable as well ($X = height$). The universe of discourse can be chosen as $U = [0, +\infty)$ (with the unit of centimeters). In this case, the fuzzy restriction on the values of u imposed by X may be defined as:

$$R(height) = \int_{190}^{+\infty} 1 / u \text{ when } u \geq 190, \text{ and}$$

$$R(height) =$$

$$\int_0^{190} \left[1 + ((190 - u) / 60)^2\right]^{-1} \Big/ u \text{ when } 0 < u \leq 190,$$

where the integral sign denotes the union rather than the summation. Then in the assignment equation $height = 180 : R(height)$, the compatibility of 180 cm with the restriction imposed by "height" becomes $c(180) = \mu_{R(height)}(180) = 0.97$.

To deal with the concept of a linguistic variable, it is helpful that a linguistic variable is a variable of a higher order than a fuzzy variable. For example, "height" can be seen as a linguistic variable whose values are "tall, quite tall, very tall, not tall, short, very short, not very tall and not very short, etc". Each of these is the name of a fuzzy variable. Moreover, the restriction imposed by "height" can be interpreted as the meaning of "height". A formal definition of a "linguistic variable" [24] follows:

Definition 4. A "*linguistic variable*" is characterized by a quintuple $(\chi, T(\chi), U, G, M)$ in which χ denotes the name of the variable; $T(\chi)$ denotes the term-set of χ, that is, the set of names of linguistic values of χ with each value being a fuzzy variable denoted generically by X (say, a "term") and ranging over a universe of discourse U which is associated with the base variable $u \in U$; and G is a syntactic rule (which usually has the form of a grammar) for generating the names, X, of values of χ; and M is a semantic rule for associating with each X its meaning, $M(X)$, which is a fuzzy subset of U.

The meaning of a term X denoted by $M(X)$ is subject to the restriction on the base variable u which is imposed by the fuzzy variable named X; that is, $M(X) \equiv R(X)$.

To illustrate what this means: "height" can be seen as a linguistic variable (i.e. $\chi = height$). The term-set associated with "height" may be expressed as

$$T(height) = \text{tall} + \text{very tall} + \text{not tall} + \text{more or less short} + \\ \text{quite short} + \text{not very tall and not very short} + \ldots, \text{etc.},$$

where each term is the name of a fuzzy variable in the universe of discourse (e.g. $U = [0, 300]$, with the unit of centimeter), and "+" denotes the union rather than the summation. The restriction imposed by a term, e.g. $R(tall)$, constitutes the meaning of "tall". (We shall argue other types of fuzzy logic in the next section to give a more precise meaning of a term.)

Endowed with the above given definitions, we can now define "complexity as a linguistic variable". Thus, "complexity" can be seen as the property of a variable. Specifically, the term-set associated with "complexity" can be expressed as the union of some fuzzy variables as:

$$T(complexity) = \text{low} + \text{quite low} + \text{very low} + \text{medium} + \text{high} + \\ \text{extremely high} + \text{not very low and not very high} + \ldots, \text{etc.}$$

Each term in the term-set is a fuzzy variable in a universe of discourse (e.g. $U = [0, 1]$).

Therefore, as discussed it is meaningful to characterize the degree to which the assignment equation is satisfied for each term by the compatibility of u with the restriction imposed on the term. To recognize the meaning of each fuzzy variable in the term-set (e.g. very high), we first need to find out the meaning of "high", then the meaning of "very", and then the meaning of "very high". Hereby the meaning of "high" can be found out by mathematical compatibility functions (e.g. see [30]). The meaning of "very" is provided by fuzzy operations (e.g. [23, 30]), and thereby the meaning of "very high" also becomes available (i.e. $very\,high = high^2$, where $high^2$ is defined by fuzzy operations).

In trying to obtain a more precise meaning for a fuzzy variable (e.g. "very high"), one may end up not finding an exact mathematical function as the compatibility function for the meaning of the fuzzy variable. In this still open case, two important attempts are in progress and will be pointed out in the final section.

4. "Complex Models" or "Complex Systems"?

It is worth emphasizing that a linguistic variable (e.g. "complexity") is always associated with two rules: (i) a syntactic rule which may have the form of a grammar that is needed to generate the names of the values of the variable, and (ii) a semantic rule which defines an algorithmic procedure that is needed to compute the meaning of each value. Apart from this, it is customary to use the adjective "complex" for "systems", e.g., the human brain is a complex system. In the present approach, however, we distinguish a "system" from the corresponding "models". Here a "system" is only a phenomenon which we encounter with and the only thing that we may see is some of its behaviors. To interpret a behavior, or system, we need models as crutches to help us in our evaluating. In other words, a system is completely silent and only by its models it begins to speak.

These models now come in various forms. They can be ordinary differential equations, partial differential equations, stochastic differential equations, fuzzy differential equations, simulation results, computer programs and so on. In addition, they can be expressed by natural languages and then would be imprecise and fuzzy. This is not a limitation of natural languages, however, as some authors have expressed (e.g. see [1]). On the contrary, the models become too powerful when we employ fuzzy logic to compute with natural language as we tried to show. Thus, the adjective "complex" appears appropriate for "models" rather than for "systems". This facon de parlance lets us say that we have "complex models" rather than "complex systems" as we do not know anything about systems without any model. For instance, a galaxy or the cosmos could be seen as a system. But we do not know anything about the system in question as long as we do not have a model. Models, however, can be made or designed from different points of view. For example, the cosmos' behavior can be modeled based upon deterministic Newtonian principles but also on Chandrasekhar's stochastic differential equations (that have yet to be understood deeply [17]). Although we encounter one phenomenon or system, we can have many models from different points of view interpreting it. In this vein it is meaningful to compare the complexity of models with each other and say, for example, the Chandrasekhar model is much more complex than a deterministic Newtonian model.

More importantly, perhaps, we believe that it is "models" which deserve the adjective "complex" rather than "systems". We here do not restrict ourselves to traditional mathematical models like, e.g., ordinary or partial differential/difference equations, etc. In other words, by "model" we mean traditional mathematical models, computer programs (as Wolfram has pointed out deeply [19]) and even fuzzy models based on natural languages [21]. Meanwhile, all measurement tools (such as those used to measure complexity as difficulty of description, e.g. entropy and algorithmic complexity; or else as difficulty of creation, e.g. time computational complexity, or as degree of organization, e.g. fractal dimension and algorithmic mutual information [11], each from a different aspect), remain valid. Nonetheless, they are all capable of being reformulated in terms of the fuzzy logic approach (e.g. see [13, 18]).

The difference between various points of view to measure the complexity of a model amount to a different "universe of discourse" being definable: "complexity as a linguistic variable". Moreover, each framework of modeling (traditional differential equations, computer programs, fuzzy if-then models, etc) defines a particular syntactic rule. Based on the specific universe of discourse and syntactic rule chosen, it is always possible to express "complexity as a linguistic variable" and specify all the elements of the quintuple offered (see Definition 4). By defining the meaning of each variable in this definition of complexity as that of a linguistic variable, it is possible to compare the models and their complexity degrees with each other based on natural languages.

We must emphasize, however, that we do not consider complexity as "difficulty of modeling"; for the concept of "difficulty" in modeling is unclear and undefined. On the other hand, one can in principle find a model by metaphysical methods (e.g. by dreaming the ring shape of the benzene molecule as Kekulé's did with his alleged dream of a snake seizing its own tail). It would be unreasonable to conclude in a specific case that since no difficulty was encountered in modeling, there is no complexity of the model. By contrast, each model has a complexity degree and we are able to compute the latter from different aspects.

5. Discussion and Next Futures

The "NeXT" computer was a joint breakthrough achieved by Stephen Wolfram and Steve Jobs 30 years ago. This mental association builds a "fuzzy" connection to our context. A new approach to considering "complexity as a linguistic variable" was suggested after reviewing some important definitions of complexity. The proposed point of view is based on the fuzzy logic approach in which everything is or is allowed to be graduated and granulated. It was shown that the adjective "complex" can be used only for models rather than for systems, and that we can compare the complexity degrees of models with each other. Under "model" we understand traditional mathematical models, computer programs, and also fuzzy models which are based on natural languages. It was discussed that to find a meaning for a given variable of a given linguistic variable, the compatibility function or the membership function plays the pivotal role.

It was found that an appropriate meaning for a variable is automatically fuzzy itself and may not be expressed as a precise mathematical function. In this regard, other types of fuzzy logic (e.g. type-2 [9, 10] in which the grade of memberships are fuzzy itself, and also the extended fuzzy logic [26] which is an attempt to find the membership functions in an uncertainty environment with imprecise and incomplete information) could acquire significance. Formulating natural languages can also be carried out by means of other types of fuzzy logic (e.g. type-2). The proposed approach is potentially important in order to find a more precise model of uncertainty and incompleteness of information [9, 10]. Other types of fuzzy logic do not weaken our new consideration of complexity but rather will help to obtain more flexible models of natural languages.

To conclude, we tried to point out one should no longer see too great a similarity between "complexity as a linguistic variable" and "complexity as difficulty of modeling" if the later definition is seen as a language property. Although the second which appears to be the currently dominant one is important, we did not accept it as a definition of complexity because the concept of difficulty does not meet clear criteria. The brief description of fuzzy logic applied to our context could show that "difficulty of modeling" does not affect our judgment to compare models with each other. Furthermore, the subjective character of the notion leads one to conclude that there is no complexity theory. By contrast, "complexity as a linguistic variable" – as a property of models rather than systems – allows one to arrive at a complexity theory that permits one to compare the complexity degrees of some given models with each other. All the complexity measurement tools stemming from different areas remain valid when applied to models and they can be reformulated in the fuzzy logic approach.

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